A Natural Proof System for Natural Language

Lecture 2: Natural Tableau System

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Previous lecture

Solve Natural Language Inference:
Given $P_1,\ldots,P_n$ premises and a hypothesis H, predict one of `entailment`, `contradiction`, and `neutral` relations.

An idea behind a tableau method:
Build a tableau by applying rules to initial signed nodes

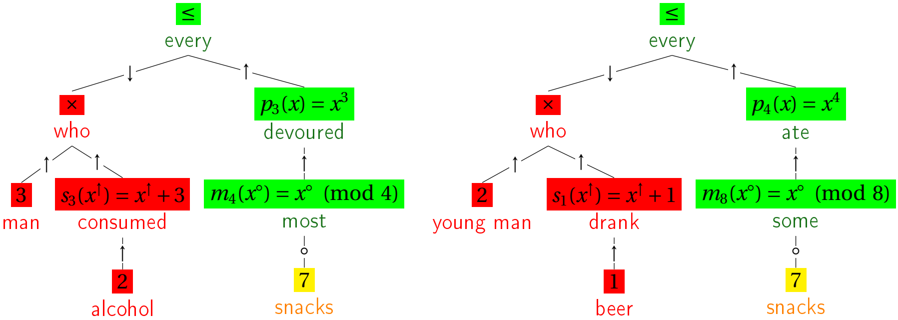
- To show $P_1,\ldots,P_n$ entails H, start a tableau with a counterexample $\{P_1:\mathbb{T},\ldots,P_n:\mathbb{T},H:\mathbb{F}\}$
- To show $P_1,\ldots,P_n$ contradicts H, start a tableau with a counterexample $\{P_1:\mathbb{T},\ldots,P_n:\mathbb{T},H:\mathbb{T}\}$

Monotonicity reasoning

GOLD: `entailment`

P: $3 \times [s_3(x) = x + 3](2) \;\le\; [p_3(x) = x^3]\big([m_4(x) = x \pmod 4](7)\big)$

H: $2 \times [s_1(x) = x + 1](1) \;\le\; [p_4(x) = x^4]\big([m_8(x) = x \pmod 8](7)\big)$



GOLD: `entailment`

P: Every man who consumed alcohol devoured most snacks

H: Every young man who drank beer ate some snacks

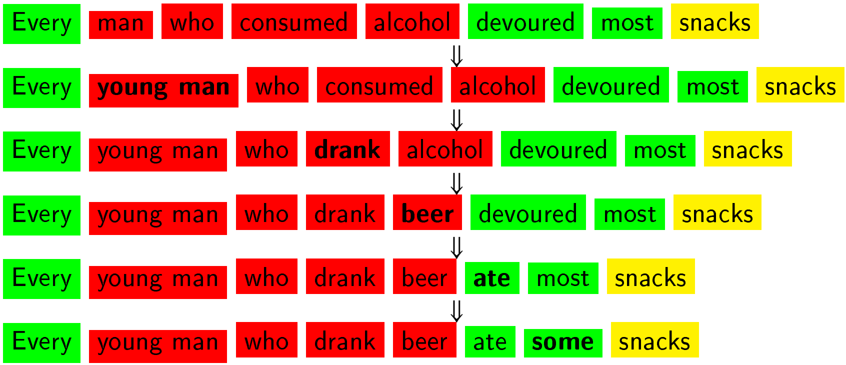
Monotonicity reasoning (II)

GOLD: `entailment`

P: Every man who consumed alcohol devoured most snacks

H: Every young man who drank beer ate some snacks

Every man who consumed alcohol devoured most snacks



Every man who consumed alcohol devoured most snacks

⇓

Every **young man** who consumed alcohol devoured most snacks

⇓

Every young man who **drank** alcohol devoured most snacks

⇓

Every young man who drank **beer** devoured most snacks

⇓

Every young man who drank beer **ate** most snacks

⇓

Every young man who drank beer ate **some** snacks

Every young man who drank beer ate some snacks

When monotonicity reasoning fails

GOLD: `entailment`
P: Not all birds fly

H: Some bird does not fly

It is impossible to prove the entailment using the monotonicity reasoning.

Today's lecture

Online demo: http://naturallogic.pro

Introduce a tableau method for natural logic, called natural tableau.

The natural tableau is an underlying theory of the online system.

Lambda Logical Forms (LLFs)

Simple type theory (with e and t basic types) is disguised as Natural Logic [Muskens, 2010].

Lambda Logical Forms are terms of the simply typed lambda calculus:

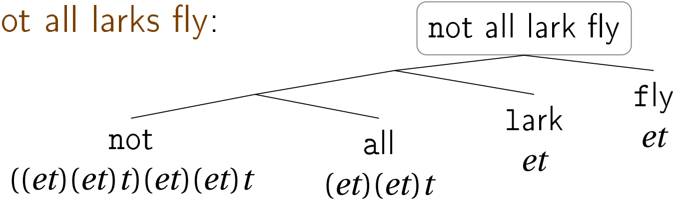
- with no logical constants
- abstraction is only over variables of type e

Examples of LLFs:

1. `((a woman)walk)`
2. `((if((a woman)walk))((no man)talk))`
3. $\texttt{(few man)}\lambda x.\texttt{(most woman)}\lambda y.\texttt{like}\,xy$

Zooming in on LLFs

Not all larks fly:



not all lark fly

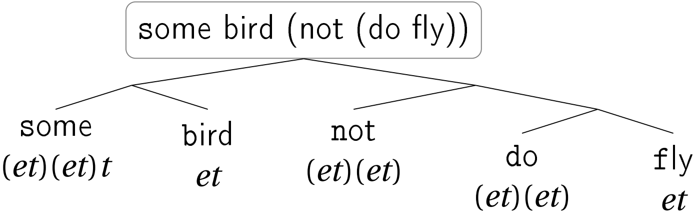
not $((et)(et)t)(et)(et)t$

all $(et)(et)t$

lark et

fly et

Some bird does not fly:



some bird (not (do fly))

some $(et)(et)t$

bird et

not $(et)(et)$

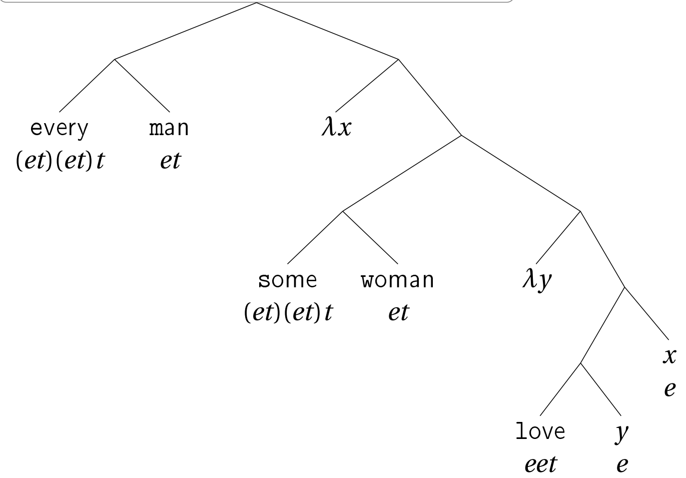
do $(et)(et)$

fly et

Zooming in on LLFs (scope ambiguity)

Every man loves some woman:

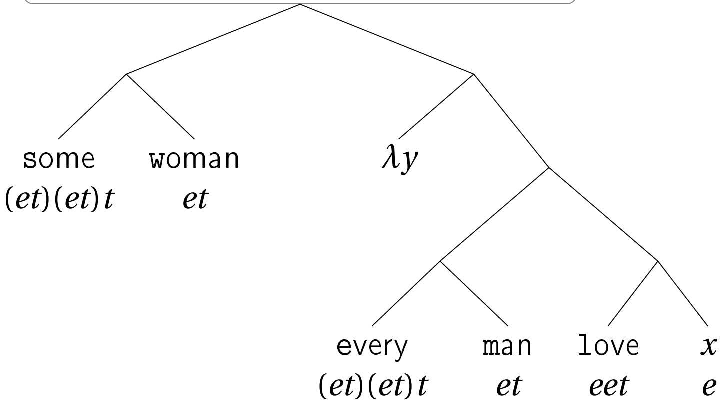
every man $(\lambda x.$ some woman $(\lambda y.$ love $y\ x))$



Zooming in on LLFs (scope ambiguity)

Every man loves some woman:

some woman $(\lambda y.$ every man (love y))



LLFs & tableau entries

LLFs are represented in tableau entries as:

$$\underbrace{\texttt{LLF:argumentList}}_{\text{Binary format of a term}}\texttt{:truthSign}$$

The `argumentList` allows traverse through and aligning shared arguments terms:

$$\texttt{no:[little bird, fly]:}\mathbb{T}$$

$$\texttt{some (littlebird) fly:[]:}\mathbb{T}$$

$$\texttt{some (little bird):[fly]:}\mathbb{T}$$

$$\texttt{some:[little bird, fly]:}\mathbb{T}$$

Ordering over the terms

We have two truth values 1 (*true*) and 0 (*false*).

With the help of $0 < 1$, we can have (partial) ordering over the terms of type $\vec{\alpha}\, t$:

- $\texttt{dog}_{et}$ is more specific than $\texttt{animal}_{et}$, because for any x, $\texttt{dog}\ x$ is less than or equal to $\texttt{animal}\ x$
- $\texttt{kiss}_{eet}$ is more specific than $\texttt{touch}_{eet}$, because for any x, y, $\texttt{kiss}\ x\ y$ is less than or equal to $\texttt{touch}\ x\ y$
- For $A_{\vec{\alpha}\, t}$ and $B_{\vec{\alpha}\, t}$, we define $A \sqsubseteq B \overset{\text{def}}{=} \forall \vec{X}.\ A\vec{X} \leq B\vec{X}$

Tableau rules in action

A>

$$\frac{A\ B : [\vec{C}] : \mathbb{X}}{A : [B, \vec{C}] : \mathbb{X}}$$

A<

$$\frac{A : [B, \vec{C}] : \mathbb{X}}{A\ B : [\vec{C}] : \mathbb{X}}$$

¬

$$\frac{\texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X}}{A : [\vec{C}] : \overline{\mathbb{X}}}$$

$\exists^{c}_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \qquad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

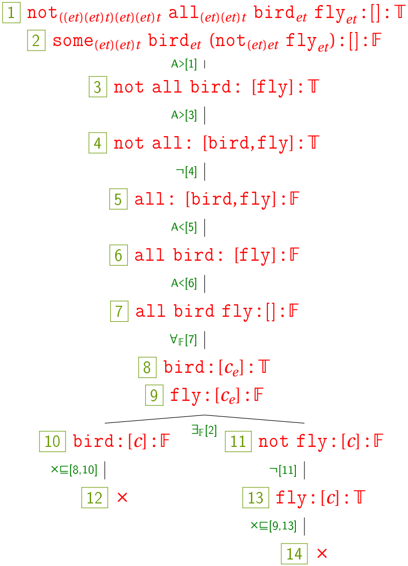
$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

$\times\sqsubseteq$

$$\frac{\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$



$\boxed{1}$ $\texttt{not}_{((et)(et)t)(et)(et)t}\ \texttt{all}_{(et)(et)t}\ \texttt{bird}_{et}\ \texttt{fly}_{et} : [\,] : \mathbb{T}$

$\boxed{2}$ $\texttt{some}_{(et)(et)t}\ \texttt{bird}_{et}\ (\texttt{not}_{(et)et}\ \texttt{fly}_{et}) : [\,] : \mathbb{F}$

A>[1]

$\boxed{3}$ $\texttt{not all bird} : [\texttt{fly}] : \mathbb{T}$

A>[3]

$\boxed{4}$ $\texttt{not all} : [\texttt{bird,fly}] : \mathbb{T}$

¬[4]

$\boxed{5}$ $\texttt{all} : [\texttt{bird,fly}] : \mathbb{F}$

A<[5]

$\boxed{6}$ $\texttt{all bird} : [\texttt{fly}] : \mathbb{F}$

A<[6]

$\boxed{7}$ $\texttt{all bird fly} : [\,] : \mathbb{F}$

$\forall_{\mathbb{F}}[7]$

$\boxed{8}$ $\texttt{bird} : [c_e] : \mathbb{T}$

$\boxed{9}$ $\texttt{fly} : [c_e] : \mathbb{F}$

$\exists_{\mathbb{F}}[2]$

$\boxed{10}$ $\texttt{bird} : [c] : \mathbb{F}$

$\times\sqsubseteq[8,10]$

$\boxed{12}$ $\times$

$\boxed{11}$ $\texttt{not fly} : [c] : \mathbb{F}$

¬[11]

$\boxed{13}$ $\texttt{fly} : [c] : \mathbb{T}$

$\times\sqsubseteq[9,13]$

$\boxed{14}$ $\times$

Monotonicity rules (Upward)

Definition (Upward monotonicity)

A function term F is upward monotone ($\uparrow$), denoted as $F^{\uparrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)\big)$$

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H^{\uparrow}\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [A, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [A, \vec{C}] : \mathbb{F} \end{array}$$

Monotonicity rules (Downward)

Definition (Downward monotonicity)

A function term F is downward monotone (↓), denoted as $F^{\downarrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \to (FY \sqsubseteq FX)\big)$$

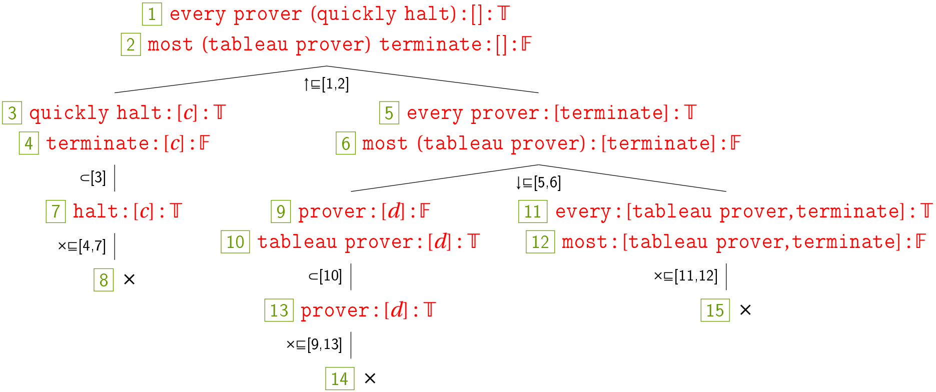
$\downarrow\sqsubseteq$

$$\begin{array}{c|c}
\multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\
\multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\
\hline
A : [\vec{D}] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\
B : [\vec{D}] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F}
\end{array}$$

$\downarrow\sqsubseteq$

$$\begin{array}{c|c}
\multicolumn{2}{c}{G\ A : [\vec{C}] : \mathbb{T}} \\
\multicolumn{2}{c}{H^{\downarrow}\ B : [\vec{C}] : \mathbb{F}} \\
\hline
A : [\vec{D}] : \mathbb{F} & G : [A, \vec{C}] : \mathbb{T} \\
B : [\vec{D}] : \mathbb{T} & H : [A, \vec{C}] : \mathbb{F}
\end{array}$$

Monotonicity rules in action



1 every prover (quickly halt) : [] : $\mathbb{T}$

2 most (tableau prover) terminate : [] : $\mathbb{F}$

↑⊑[1,2]

3 quickly halt : [c] : $\mathbb{T}$

4 terminate : [c] : $\mathbb{F}$

⊂[3]

7 halt : [c] : $\mathbb{T}$

×⊑[4,7]

8 ×

5 every prover : [terminate] : $\mathbb{T}$

6 most (tableau prover) : [terminate] : $\mathbb{F}$

↓⊑[5,6]

9 prover : [d] : $\mathbb{F}$

10 tableau prover : [d] : $\mathbb{T}$

⊂[10]

13 prover : [d] : $\mathbb{T}$

×⊑[9,13]

14 ×

11 every : [tableau prover, terminate] : $\mathbb{T}$

12 most : [tableau prover, terminate] : $\mathbb{F}$

×⊑[11,12]

15 ×

↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}\,] : \mathbb{T} \quad B : [\vec{d}\,] : \mathbb{F} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T} \quad H : [B, \vec{C}] : \mathbb{F}}$$

↓⊑

$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}\,] : \mathbb{F} \quad B : [\vec{d}\,] : \mathbb{T} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T} \quad H : [B, \vec{C}] : \mathbb{F}}$$

⊂

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

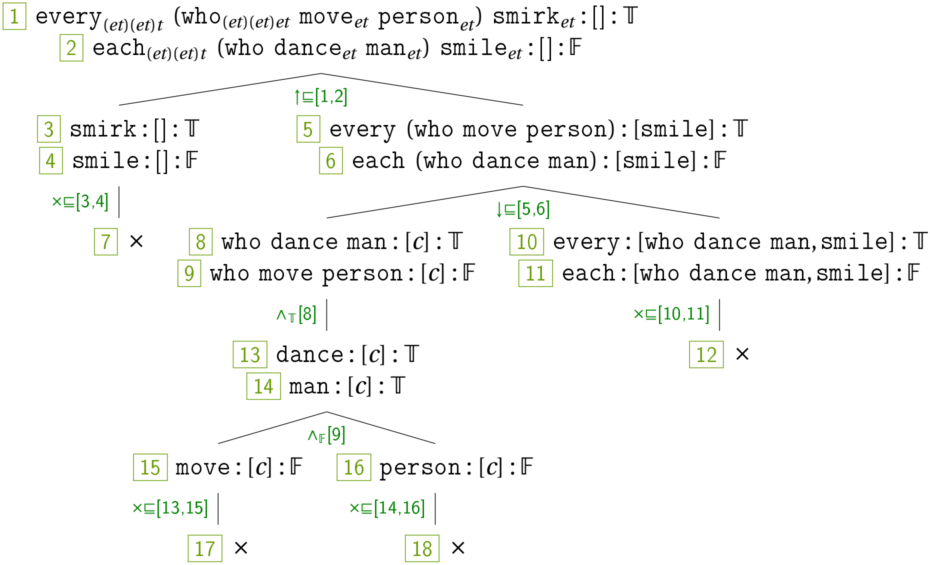
$$\forall X (A^{\subset} X \sqsubseteq X)$$

×⊑

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

$A \sqsubseteq B$

Monotonicity rules in action (II)



$\boxed{1}\ \texttt{every}_{(et)(et)t}\ (\texttt{who}_{(et)(et)et}\ \texttt{move}_{et}\ \texttt{person}_{et})\ \texttt{smirk}_{et} : [\,] : \mathbb{T}$

$\boxed{2}\ \texttt{each}_{(et)(et)t}\ (\texttt{who}\ \texttt{dance}_{et}\ \texttt{man}_{et})\ \texttt{smile}_{et} : [\,] : \mathbb{F}$

$\uparrow\sqsubseteq[1,2]$

$\boxed{3}\ \texttt{smirk} : [\,] : \mathbb{T}$

$\boxed{4}\ \texttt{smile} : [\,] : \mathbb{F}$

$\boxed{5}\ \texttt{every}\ (\texttt{who move person}) : [\texttt{smile}] : \mathbb{T}$

$\boxed{6}\ \texttt{each}\ (\texttt{who dance man}) : [\texttt{smile}] : \mathbb{F}$

$\times\sqsubseteq[3,4]$

$\boxed{7}\ \times$

$\downarrow\sqsubseteq[5,6]$

$\boxed{8}\ \texttt{who dance man} : [c] : \mathbb{T}$

$\boxed{9}\ \texttt{who move person} : [c] : \mathbb{F}$

$\boxed{10}\ \texttt{every} : [\texttt{who dance man}, \texttt{smile}] : \mathbb{T}$

$\boxed{11}\ \texttt{each} : [\texttt{who dance man}, \texttt{smile}] : \mathbb{F}$

$\wedge_{\mathbb{T}}[8]$

$\boxed{13}\ \texttt{dance} : [c] : \mathbb{T}$

$\boxed{14}\ \texttt{man} : [c] : \mathbb{T}$

$\times\sqsubseteq[10,11]$

$\boxed{12}\ \times$

$\wedge_{\mathbb{F}}[9]$

$\boxed{15}\ \texttt{move} : [c] : \mathbb{F}$

$\boxed{16}\ \texttt{person} : [c] : \mathbb{F}$

$\times\sqsubseteq[13,15]$

$\boxed{17}\ \times$

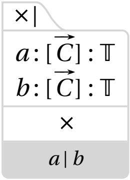
$\times\sqsubseteq[14,16]$

$\boxed{18}\ \times$

Semantic exclusion and exhaustion

Definition (Exclusion)

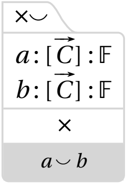
$A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are in an *exclusion* relation iff they are not true on the same arguments.



$$\frac{\begin{array}{c} a : [\vec{C}] : \mathbb{T} \\ b : [\vec{C}] : \mathbb{T} \end{array}}{\times} \; {\times}| \quad a \,|\, b$$

Definition (Joint exhaustion)

$A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are *jointly exhaustive* iff for any argument at least one of them is true on it. $A \smile B \overset{\text{def}}{=} \forall \vec{X}.(A \sqcup B)\vec{X}$

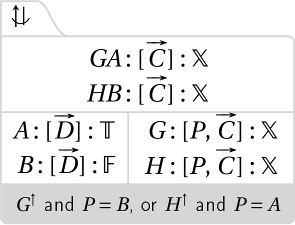


$$\frac{\begin{array}{c} a : [\vec{C}] : \mathbb{F} \\ b : [\vec{C}] : \mathbb{F} \end{array}}{\times} \; {\times}\smile \quad a \smile b$$

(**dog** | **cat**), (**many** | **few**), (**sleep** | **run**)

(**nonhuman** ⌣ **animal**), (**at least six** ⌣ **at most ten**)

Semantic exclusion and exhaustion (II)



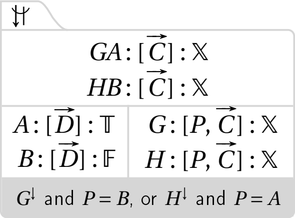
⥮

$GA : [\vec{C}] : \mathbb{X}$

$HB : [\vec{C}] : \mathbb{X}$

$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$	$G : [P, \vec{C}] : \mathbb{X}$ $H : [P, \vec{C}] : \mathbb{X}$

$G^{\upharpoonleft}$ and $P = B$, or $H^{\upharpoonleft}$ and $P = A$



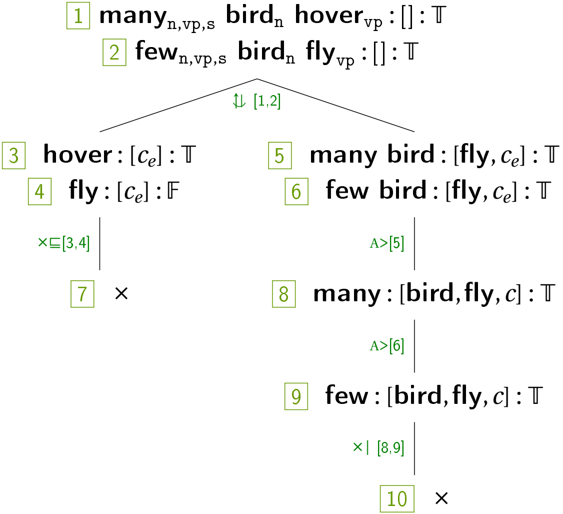
⥯

$GA : [\vec{C}] : \mathbb{X}$

$HB : [\vec{C}] : \mathbb{X}$

$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$	$G : [P, \vec{C}] : \mathbb{X}$ $H : [P, \vec{C}] : \mathbb{X}$

$G^{\downharpoonleft}$ and $P = B$, or $H^{\downharpoonleft}$ and $P = A$



1 $\textbf{many}_{\mathtt{n,vp,s}}$ $\textbf{bird}_{\mathtt{n}}$ $\textbf{hover}_{\mathtt{vp}} : [\,] : \mathbb{T}$

2 $\textbf{few}_{\mathtt{n,vp,s}}$ $\textbf{bird}_{\mathtt{n}}$ $\textbf{fly}_{\mathtt{vp}} : [\,] : \mathbb{T}$

⥮ [1,2]

3 **hover** $: [c_e] : \mathbb{T}$

4 **fly** $: [c_e] : \mathbb{F}$

×⊑[3,4]

7 ×

5 **many bird** $: [$**fly**$, c_e] : \mathbb{T}$

6 **few bird** $: [$**fly**$, c_e] : \mathbb{T}$

A>[5]

8 **many** $: [$**bird**, **fly**$, c] : \mathbb{T}$

A>[6]

9 **few** $: [$**bird**, **fly**$, c] : \mathbb{T}$

×| [8,9]

10 ×

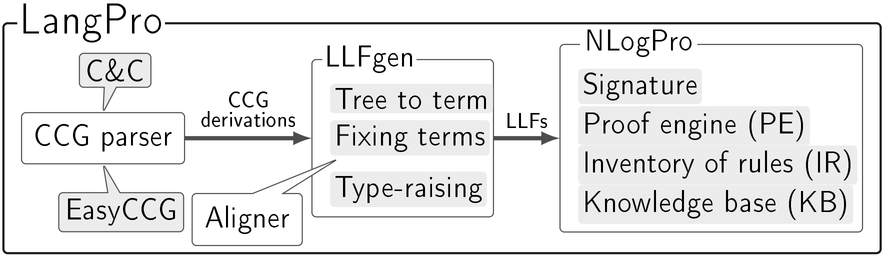
Conclusion

Simple type theory as a proxy to natural logic

Tableau system for natural logic

Monotonicity reasoning enabled by the monotone rules

Tomorrow


LangPro
C&C
CCG parser
EasyCCG
CCG derivations
Aligner
LLFgen
Tree to term
Fixing terms
Type-raising
LLFs
NLogPro
Signature
Proof engine (PE)
Inventory of rules (IR)
Knowledge base (KB)

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