A Natural Proof System for Natural Language

NPS4NL-3: Natural Tableau System

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Where are we now?

Last two lectures:

- The goal is to tackle the NLI problem with an explainable model
- We learned how a tableau method works
- We know a little of *simple* type theory

Today's lecture:

- Introduce a tableau method for natural logic [Muskens, 2010]
- Further tune it for the NLI problem

Background

- In traditional, pre-Fregean, logic, Latin and other languages have always been the main vehicle of representation.
- Frege's revolution was a sharp break with this tradition. Ordinary language was thought to be too vague to be amenable to scientific treatment, let alone to be the basis of logic itself.
- But in the 1960s this all changed again with Richard Montague, who showed how fragments of English can be treated in essentially Fregean ways.
- Montague wrote that there are no important theoretical differences between natural languages and logical languages.

Aim

- But if there are no important theoretical differences between natural languages and logical languages, it should be possible to find a calculus for the entailment relation in natural language that uses only linguistic forms (or, perhaps, only forms in a language of thought).
- This calculus should also use only rules that are linguistically relevant.
- And ideally it should be possible to say more about natural reasoning with the help of such a logic.
- This brings us to natural logic, the continuation of traditional logic with modern means.
- My aim is to bring such a logic closer to realization by working on a tableau calculus for linguistic representations.

Why Tableaus?

- One answer is that tableaus model the search for verifying models that Johnson-Laird and his co-workers have been advocating as a model of interpretation in the psychology of reasoning. I find the picture of interpretation as search for a verifying model appealing.
- (The proximity between tableau methods and the aims of Johnson-Laird's mental models theory has been emphasized in Allan Ramsay's work.)
- Tableaus potentially can model various modes of reasoning:
    - classical reasoning
    - reasoning on the basis of minimal models / closed world assumption (Olivetti, . . .)
    - abduction (Cialdea Mayer & Pirri, Aliseda, . . .)

Lambda Logical Forms (LLFs)

Simple type theory (with e and t basic types)$*$ is disguised as Natural Logic.

We consider terms of the simply typed lambda calculus, called Lambda Logical Forms, in which no logical constants occur and in which abstraction is only over variables of type e.

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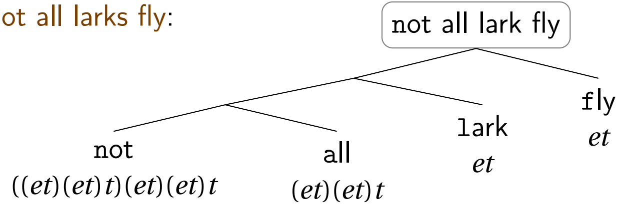
Examples of LLFs:

1. ((a woman)walk)
2. ((if((a woman)walk))((no man)talk))
3. (mary(think((if((a woman)walk))((no man)talk))))
4. ((a woman)(λx(mary(think((if(walk x))((no man)talk))))))
5. (few man)$\lambda x.$(most woman)$\lambda y.$like xy
6. (mary λx((try(run x)) x))

Zooming in on LLFs

Not all larks fly:



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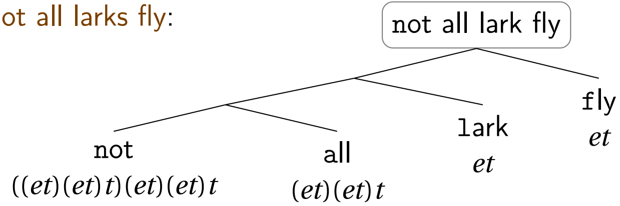


not all lark fly

not

$((et)(et)t)(et)(et)t$

all

$(et)(et)t$

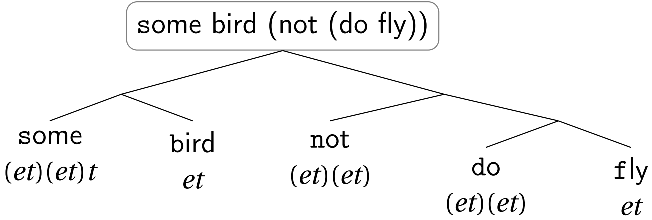
lark

et

fly

et

Some bird does not fly:



some bird (not (do fly))

some

$(et)(et)t$

bird

et

not

$(et)(et)$

do

$(et)(et)$

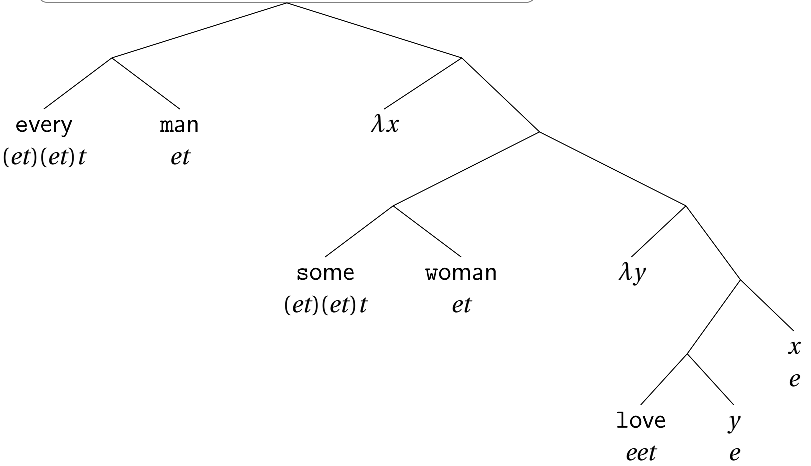
fly

et

Zooming in on LLFs (scope ambiguity)

Every man loves some woman:

every man (λx. some woman (λy. love y x))



Zooming in on LLFs (scope ambiguity)

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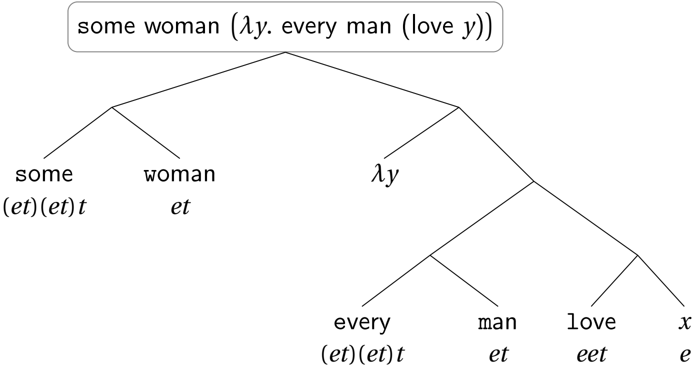

some woman (λy. every man (love y))

some $(et)(et)t$

woman et

λy

every $(et)(et)t$

man et

love eet

x e

LLFs & tableau entries

LLFs are represented in tableau entries as:

$$\underbrace{\texttt{LLF} : \texttt{argumentList}}_{\text{Binary format of a term}} : \texttt{truthSign}$$

Different binary representations of the same signed term:

$\texttt{love john} : [\texttt{mary}] : \mathbb{F}$ $\quad$ $\texttt{love} : [\texttt{john,mary}] : \mathbb{F}$

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Two advantages of `argumentList`:

- Traverse through a recursive structure of a term
- Align the shared arguments and contrast different terms:

$$\texttt{no}:[\texttt{little bird},\ \texttt{fly}]:\mathbb{T}$$
$$\texttt{some (littlebird) fly}:[\,]:\mathbb{T}$$

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$$\texttt{no} : [\texttt{little bird, fly}] : \mathbb{T}$$
$$\texttt{some (littlebird) fly} : [] : \mathbb{T}$$
$$\texttt{some (little bird)} : [\texttt{fly}] : \mathbb{T}$$

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`love john : [mary] : 𝔽`   `love : [john,mary] : 𝔽`

Two advantages of `argumentList`:

- Traverse through a recursive structure of a term
- Align the shared arguments and contrast different terms:

`no : [little bird, fly] : 𝕋`
`some (littlebird) fly : [] : 𝕋`
`some (little bird) : [fly] : 𝕋`
`some : [little bird, fly] : 𝕋`

Ordering over the terms

Remember we use types built up from e and t.

We have two truth values 1 (*true*) and 0 (*false*), where $0 < 1$.

With the help of $0 < 1$, we can have (partial) ordering over the terms of type $\vec{\alpha}\,t$:

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Sequence/vector notation conventions:

- $\alpha_1 \ldots \alpha_n t \equiv \vec{\alpha}\,t$
- $A B_1 B_2 \ldots B_n \equiv A\vec{B}$

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- $\texttt{dog}_{et}$ is more specific than $\texttt{animal}_{et}$, because for any x, $\texttt{dog}\ x$ is less than or equal to $\texttt{animal}\ x$

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- $\texttt{dog}_{et}$ is more specific than $\texttt{animal}_{et}$, because for any x, $\texttt{dog}\ x$ is less than or equal to $\texttt{animal}\ x$
- $\texttt{kiss}_{eet}$ is more specific than $\texttt{touch}_{eet}$, because for any x, y, $\texttt{kiss}\ x\ y$ is less than or equal to $\texttt{animal}\ x\ y$

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We have two truth values 1 (*true*) and 0 (*false*), where $0 < 1$.

With the help of $0 < 1$, we can have (partial) ordering over the terms of type $\vec{\alpha}\, t$:

- `dog`$_{et}$ is more specific than `animal`$_{et}$, because for any x, `dog` x is less than or equal to `animal` x
- `kiss`$_{eet}$ is more specific than `touch`$_{eet}$, because for any x, y, `kiss` $x\ y$ is less than or equal to `animal` $x\ y$
- For $A_{\vec{\alpha}\,t}$ and $B_{\vec{\alpha}\,t}$, we define $A \sqsubseteq B \overset{\text{def}}{=} \forall \vec{X}.\ A\vec{X} \leq B\vec{X}$

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- $\alpha_1 \ldots \alpha_n t \equiv \vec{\alpha}\, t$
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Some of the tableau rules

$$\begin{array}{|c|}\hline \text{A>} \\ \hline A\ B : [\vec{C}] : \mathbb{X} \\ \hline A : [B, \vec{C}] : \mathbb{X} \\ \hline \end{array} \qquad \begin{array}{|c|}\hline \text{A<} \\ \hline A : [B, \vec{C}] : \mathbb{X} \\ \hline A\ B : [\vec{C}] : \mathbb{X} \\ \hline \end{array} \qquad \begin{array}{|c|}\hline \times\sqsubseteq \\ \hline A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \\ \hline \end{array} \qquad \begin{array}{|c|}\hline \neg \\ \hline \texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X} \\ \hline A : [\vec{C}] : \overline{\mathbb{X}} \\ \hline \end{array}$$

Some of the tableau rules

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$$\begin{array}{c} A\ B : [\vec{C}] : \mathbb{X} \\ \hline A : [B, \vec{C}] : \mathbb{X} \end{array}$$

A<

$$\begin{array}{c} A : [B, \vec{C}] : \mathbb{X} \\ \hline A\ B : [\vec{C}] : \mathbb{X} \end{array}$$

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

¬

$$\begin{array}{c} \texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X} \\ \hline A : [\vec{C}] : \overline{\mathbb{X}} \end{array}$$

$\wedge_{\mathbb{T}}$

$$\begin{array}{c} F_{\alpha\alpha\alpha} A\ B : [\vec{C}] : \mathbb{T} \\ \hline A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{T} \\ \hline F \in \{\texttt{and}, \texttt{who}\} \end{array}$$

$\wedge_{\mathbb{F}}$

$$\begin{array}{c|c} \multicolumn{2}{c}{F_{\alpha\alpha\alpha} A\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{C}] : \mathbb{F} & B : [\vec{C}] : \mathbb{F} \\ \hline \multicolumn{2}{c}{F \in \{\texttt{and}, \texttt{who}\}} \end{array}$$

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$$\begin{array}{c} \texttt{or}_{\alpha\alpha\alpha} A\ B : [\vec{C}] : \mathbb{F} \\ \hline A : [\vec{C}] : \mathbb{F} \\ B : [\vec{C}] : \mathbb{F} \end{array}$$

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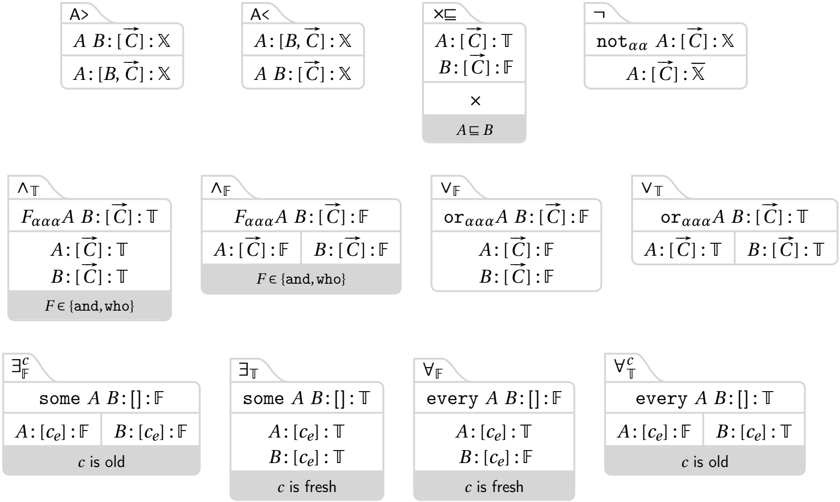
$$\begin{array}{c|c} \multicolumn{2}{c}{\texttt{or}_{\alpha\alpha\alpha} A\ B : [\vec{C}] : \mathbb{T}} \\ \hline A : [\vec{C}] : \mathbb{T} & B : [\vec{C}] : \mathbb{T} \end{array}$$

Some of the tableau rules



A>

$$A\ B : [\vec{C}] : \mathbb{X}$$
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A<

$$A : [B, \vec{C}] : \mathbb{X}$$
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×⊑

$$A : [\vec{C}] : \mathbb{T}$$
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$$\times$$

$A \sqsubseteq B$

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$$F_{\alpha\alpha\alpha} A\ B : [\vec{C}] : \mathbb{F}$$
$$A : [\vec{C}] : \mathbb{F} \quad | \quad B : [\vec{C}] : \mathbb{F}$$

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$\exists^{c}_{\mathbb{F}}$

$$\texttt{some}\ A\ B : [\,] : \mathbb{F}$$
$$A : [c_e] : \mathbb{F} \quad | \quad B : [c_e] : \mathbb{F}$$

c is old

$\exists_{\mathbb{T}}$

$$\texttt{some}\ A\ B : [\,] : \mathbb{T}$$
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c is fresh

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Tableau rules in action

$$\text{A>}\quad \frac{A\ B : [\vec{C}] : \mathbb{X}}{A : [B, \vec{C}] : \mathbb{X}}$$

$$\text{A<}\quad \frac{A : [B, \vec{C}] : \mathbb{X}}{A\ B : [\vec{C}] : \mathbb{X}}$$

$$\neg\quad \frac{\mathtt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X}}{A : [\vec{C}] : \overline{\mathbb{X}}}$$

$$\exists^{c}_{\mathbb{F}}\quad \frac{\mathtt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad \mid \quad B : [c_e] : \mathbb{F}}$$

c is old

$$\forall_{\mathbb{F}}\quad \frac{\mathtt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

$$\times\sqsubseteq\quad \frac{\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$

1 $\mathtt{not}_{((et)(et)t)(et)(et)t}\ \mathtt{all}_{(et)(et)t}\ \mathtt{bird}_{et}\ \mathtt{fly}_{et} : [\,] : \mathbb{T}$

2 $\mathtt{some}_{(et)(et)t}\ \mathtt{bird}_{et}\ (\mathtt{not}_{(et)et}\ \mathtt{fly}_{et}) : [\,] : \mathbb{F}$

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A>[1]

3 $\texttt{not all bird} : [\texttt{fly}] : \mathbb{T}$

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A>[3]

$\boxed{4}$ $\texttt{not all: [bird,fly]} : \mathbb{T}$

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$$\times$$

$A \sqsubseteq B$

1 $\mathtt{not}_{((et)(et)t)(et)(et)t}\ \mathtt{all}_{(et)(et)t}\ \mathtt{bird}_{et}\ \mathtt{fly}_{et} : [\,] : \mathbb{T}$

2 $\mathtt{some}_{(et)(et)t}\ \mathtt{bird}_{et}\ (\mathtt{not}_{(et)et}\ \mathtt{fly}_{et}) : [\,] : \mathbb{F}$

A>[1]

3 $\mathtt{not\ all\ bird} : [\mathtt{fly}] : \mathbb{T}$

A>[3]

4 $\mathtt{not\ all} : [\mathtt{bird}, \mathtt{fly}] : \mathbb{T}$

Tableau rules in action

A>

$A\ B : [\vec{C}] : \mathbb{X}$

$A : [B, \vec{C}] : \mathbb{X}$

A<

$A : [B, \vec{C}] : \mathbb{X}$

$A\ B : [\vec{C}] : \mathbb{X}$

¬

$\texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X}$

$A : [\vec{C}] : \overline{\mathbb{X}}$

$\exists^{c}_{\mathbb{F}}$

$\texttt{some}\ A\ B : [\,] : \mathbb{F}$

$A : [c_e] : \mathbb{F}$ | $B : [c_e] : \mathbb{F}$

c is old

$\forall_{\mathbb{F}}$

$\texttt{every}\ A\ B : [\,] : \mathbb{F}$

$A : [c_e] : \mathbb{T}$
$B : [c_e] : \mathbb{F}$

c is fresh

$\times\sqsubseteq$

$A : [\vec{C}] : \mathbb{T}$
$B : [\vec{C}] : \mathbb{F}$

$\times$

$A \sqsubseteq B$

1 $\texttt{not}_{((et)(et)t)(et)(et)t}\ \texttt{all}_{(et)(et)t}\ \texttt{bird}_{et}\ \texttt{fly}_{et} : [\,] : \mathbb{T}$

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$$\frac{A\ B : [\vec{C}] : \mathbb{X}}{A : [B, \vec{C}] : \mathbb{X}}$$

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$$\frac{\texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X}}{A : [\vec{C}] : \overline{\mathbb{X}}}$$

$\exists^{c}_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad \Big| \quad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

$\times\sqsubseteq$

$$\frac{\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$

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Tableau rules in action

A>

$$\begin{array}{c} A\;B : [\vec{C}] : \mathbb{X} \\ \hline A : [B, \vec{C}] : \mathbb{X} \end{array}$$

A<

$$\begin{array}{c} A : [B, \vec{C}] : \mathbb{X} \\ \hline A\;B : [\vec{C}] : \mathbb{X} \end{array}$$

¬

$$\begin{array}{c} \texttt{not}_{\alpha\alpha}\; A : [\vec{C}] : \mathbb{X} \\ \hline A : [\vec{C}] : \overline{\mathbb{X}} \end{array}$$

$\exists^{c}_{\mathbb{F}}$

$$\begin{array}{c|c} \multicolumn{2}{c}{\texttt{some}\; A\; B : [\,] : \mathbb{F}} \\ \hline A : [c_e] : \mathbb{F} & B : [c_e] : \mathbb{F} \end{array}$$

c is old

$\forall_{\mathbb{F}}$

$$\begin{array}{c} \texttt{every}\; A\; B : [\,] : \mathbb{F} \\ \hline A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}$$

c is fresh

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \end{array}$$

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$$\begin{array}{c} A : [B, \vec{C}] : \mathbb{X} \\ \hline A\ B : [\vec{C}] : \mathbb{X} \end{array}$$

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$$\begin{array}{c} \texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X} \\ \hline A : [\vec{C}] : \overline{\mathbb{X}} \end{array}$$

$\exists^{c}_{\mathbb{F}}$

$$\begin{array}{c|c} \multicolumn{2}{c}{\texttt{some}\ A\ B : [\,] : \mathbb{F}} \\ \hline A : [c_e] : \mathbb{F} & B : [c_e] : \mathbb{F} \end{array}$$

c is old

$\forall_{\mathbb{F}}$

$$\begin{array}{c} \texttt{every}\ A\ B : [\,] : \mathbb{F} \\ \hline A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}$$

c is fresh

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \end{array}$$

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$\exists^c_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad \mid \quad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

$\times\sqsubseteq$

$$\frac{\begin{array}{c} A : [\vec{C}\,] : \mathbb{T} \\ B : [\vec{C}\,] : \mathbb{F} \end{array}}{\times}$$

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A>[3]

$\boxed{4}$ $\texttt{not all} : [\texttt{bird}, \texttt{fly}] : \mathbb{T}$

¬[4]

$\boxed{5}$ $\texttt{all} : [\texttt{bird}, \texttt{fly}] : \mathbb{F}$

A<[5]

$\boxed{6}$ $\texttt{all bird} : [\texttt{fly}] : \mathbb{F}$

Tableau rules in action

A>

$$\frac{A\ B : [\vec{C}\,] : \mathbb{X}}{A : [B, \vec{C}\,] : \mathbb{X}}$$

A<

$$\frac{A : [B, \vec{C}\,] : \mathbb{X}}{A\ B : [\vec{C}\,] : \mathbb{X}}$$

¬

$$\frac{\texttt{not}_{\alpha\alpha}\ A : [\vec{C}\,] : \mathbb{X}}{A : [\vec{C}\,] : \overline{\mathbb{X}}}$$

$\exists^{c}_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad \mid \quad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{matrix} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{matrix}}$$

c is fresh

$\times\sqsubseteq$

$$\frac{\begin{matrix} A : [\vec{C}\,] : \mathbb{T} \\ B : [\vec{C}\,] : \mathbb{F} \end{matrix}}{\times}$$

$A \sqsubseteq B$

1 $\texttt{not}_{((et)(et)t)(et)(et)t}\ \texttt{all}_{(et)(et)t}\ \texttt{bird}_{et}\ \texttt{fly}_{et} : [\,] : \mathbb{T}$

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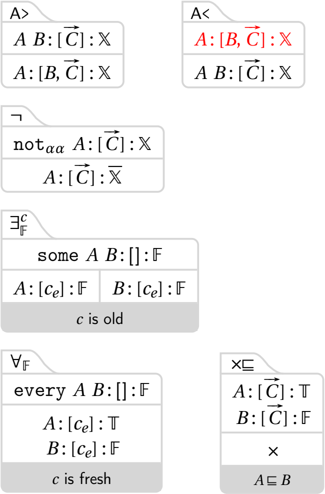
A<[5]

6 $\texttt{all bird} : [\texttt{fly}] : \mathbb{F}$

Tableau rules in action



A>

$A\ B : [\vec{C}] : \mathbb{X}$

$A : [B, \vec{C}] : \mathbb{X}$

A<

$A : [B, \vec{C}] : \mathbb{X}$

$A\ B : [\vec{C}] : \mathbb{X}$

¬

$\texttt{not}_{\alpha\alpha}\ A : [\vec{C}] : \mathbb{X}$

$A : [\vec{C}] : \overline{\mathbb{X}}$

$\exists^{c}_{\mathbb{F}}$

$\texttt{some}\ A\ B : [\,] : \mathbb{F}$

$A : [c_e] : \mathbb{F}$ | $B : [c_e] : \mathbb{F}$

c is old

$\forall_{\mathbb{F}}$

$\texttt{every}\ A\ B : [\,] : \mathbb{F}$

$A : [c_e] : \mathbb{T}$

$B : [c_e] : \mathbb{F}$

c is fresh

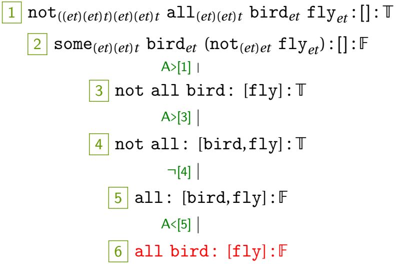
$\times\sqsubseteq$

$A : [\vec{C}] : \mathbb{T}$

$B : [\vec{C}] : \mathbb{F}$

×

$A \sqsubseteq B$



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A>[1]

$\boxed{3}$ $\texttt{not all bird: [fly]} : \mathbb{T}$

A>[3]

$\boxed{4}$ $\texttt{not all: [bird,fly]} : \mathbb{T}$

¬[4]

$\boxed{5}$ $\texttt{all: [bird,fly]} : \mathbb{F}$

A<[5]

$\boxed{6}$ $\texttt{all bird: [fly]} : \mathbb{F}$

Tableau rules in action

A>

$$\frac{A\ B : [\vec{C}] : \mathbb{X}}{A : [B, \vec{C}] : \mathbb{X}}$$

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$\exists^{c}_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad \mid \quad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

$\times\sqsubseteq$

$$\frac{\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$

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A<[5]

6 $\texttt{all bird} : [\texttt{fly}] : \mathbb{F}$

A<[6]

7 $\texttt{all bird fly} : [\,] : \mathbb{F}$

Tableau rules in action

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c is old

$\forall_{\mathbb{F}}$

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A<[5]

$\boxed{6}$ $\mathtt{all\ bird} : [\mathtt{fly}] : \mathbb{F}$

A<[6]

$\boxed{7}$ $\mathtt{all\ bird\ fly} : [\,] : \mathbb{F}$

Tableau rules in action

A>

$$\frac{A\ B : [\vec{C}] : \mathbb{X}}{A : [B, \vec{C}] : \mathbb{X}}$$

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c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

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c is old

$\forall_{\mathbb{F}}$

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$\times\sqsubseteq$

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Tableau rules in action

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$$\frac{A\ B : [\vec{C}\,] : \mathbb{X}}{A : [B, \vec{C}\,] : \mathbb{X}}$$

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$\exists^{c}_{\mathbb{F}}$

$$\frac{\texttt{some}\ A\ B : [\,] : \mathbb{F}}{A : [c_e] : \mathbb{F} \quad\mid\quad B : [c_e] : \mathbb{F}}$$

c is old

$\forall_{\mathbb{F}}$

$$\frac{\texttt{every}\ A\ B : [\,] : \mathbb{F}}{\begin{array}{c} A : [c_e] : \mathbb{T} \\ B : [c_e] : \mathbb{F} \end{array}}$$

c is fresh

×⊑

$$\frac{\begin{array}{c} A : [\vec{C}\,] : \mathbb{T} \\ B : [\vec{C}\,] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$

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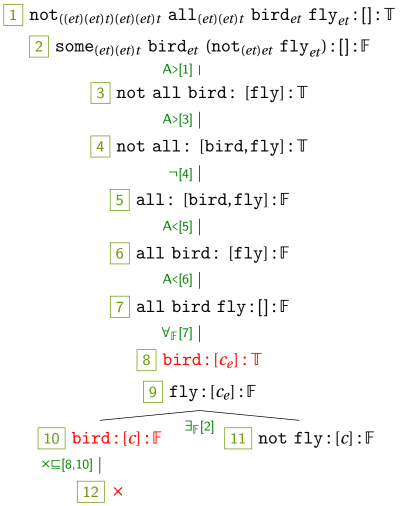

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12 ×

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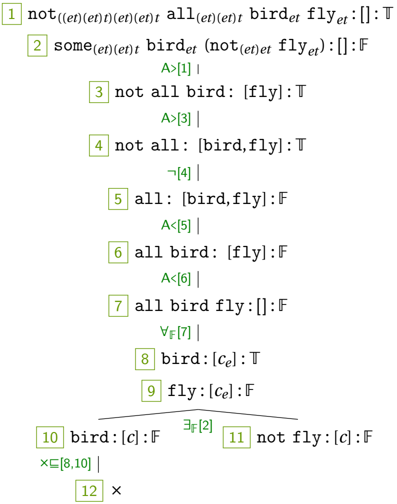

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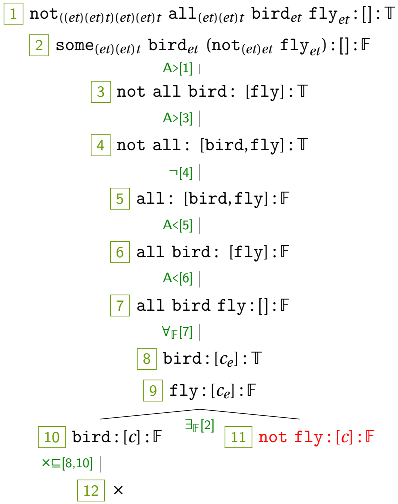
×

$A \sqsubseteq B$



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×⊑

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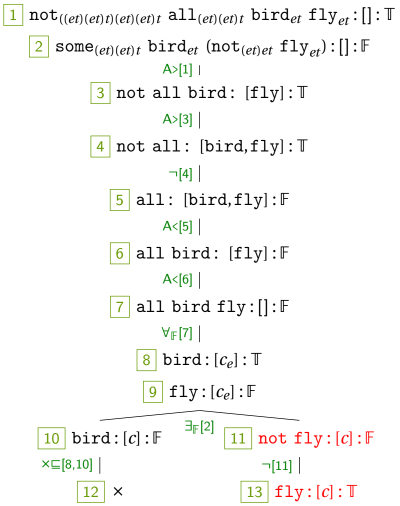
×

$A \sqsubseteq B$



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A>[1]

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A<[5]

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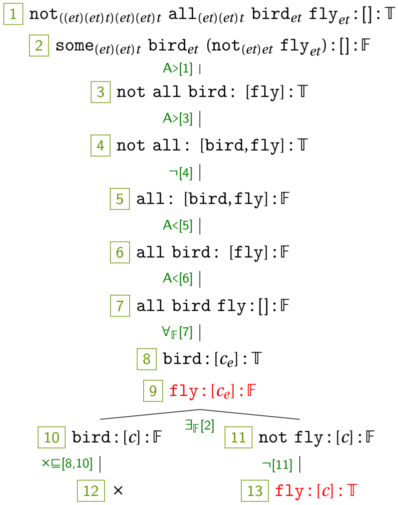

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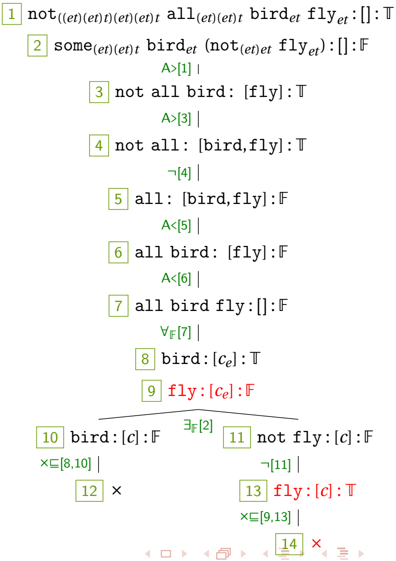

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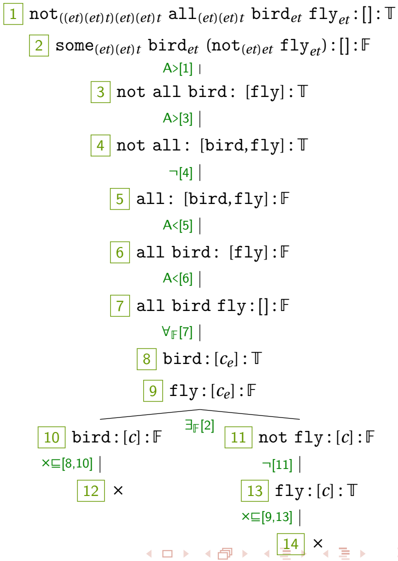

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4 $\texttt{not all} : [\texttt{bird}, \texttt{fly}] : \mathbb{T}$

¬[4]

5 $\texttt{all} : [\texttt{bird}, \texttt{fly}] : \mathbb{F}$

A<[5]

6 $\texttt{all bird} : [\texttt{fly}] : \mathbb{F}$

A<[6]

7 $\texttt{all bird fly} : [\,] : \mathbb{F}$

$\forall_{\mathbb{F}}$[7]

8 $\texttt{bird} : [c_e] : \mathbb{T}$

9 $\texttt{fly} : [c_e] : \mathbb{F}$

$\exists_{\mathbb{F}}$[2]

10 $\texttt{bird} : [c] : \mathbb{F}$

×⊑[8,10]

12 ×

11 $\texttt{not fly} : [c] : \mathbb{F}$

¬[11]

13 $\texttt{fly} : [c] : \mathbb{T}$

×⊑[9,13]

14 ×

Monotonicity rules (Upward)

Definition (Upward monotonicity)

A function term F of type $(\vec{\alpha}\,t)\vec{\gamma}\,t$ is upward monotone (↑), denoted as $F^{\uparrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \to (FX \sqsubseteq FY)\big)$$
$$\forall XY\big(F(X \sqcap Y) \sqsubseteq (FX \sqcap FY)\big)$$
$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcup Y)\big)$$

$$A_{\vec{\alpha}\,t} \sqcap B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \wedge B\vec{x} \qquad A_{\vec{\alpha}\,t} \sqcup B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \vee B\vec{x}$$

Monotonicity rules (Upward)

Definition (Upward monotonicity)

A function term F of type $(\vec{\alpha}\,t)\vec{\gamma}\,t$ is upward monotone ($\uparrow$), denoted as $F^{\uparrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \to (FX \sqsubseteq FY)\big)$$

$$\forall XY\big(F(X \sqcap Y) \sqsubseteq (FX \sqcap FY)\big)$$

$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcup Y)\big)$$

$$A_{\vec{\alpha}\,t} \sqcap B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \wedge B\vec{x} \qquad A_{\vec{\alpha}\,t} \sqcup B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \vee B\vec{x}$$

$$\begin{array}{c} \uparrow\sqsubseteq \\ G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \\ H\ B : [\vec{C}] : \mathbb{F} \\ \hline \begin{array}{c|c} A : [\vec{D}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array} \end{array} \quad + \quad \begin{array}{c} \uparrow\sqsubseteq \\ G\ A : [\vec{C}] : \mathbb{T} \\ H^{\uparrow}\ B : [\vec{C}] : \mathbb{F} \\ \hline \begin{array}{c|c} A : [\vec{D}] : \mathbb{T} & G : [A, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [A, \vec{C}] : \mathbb{F} \end{array} \end{array}$$

Monotonicity rules (Upward)

Definition (Upward monotonicity)

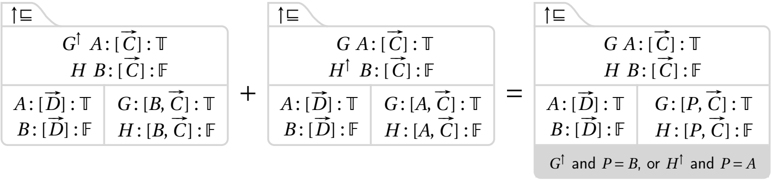
A function term F of type $(\vec{\alpha}\,t)\vec{\gamma}\,t$ is upward monotone ($\uparrow$), denoted as $F^{\uparrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)\big)$$

$$\forall XY\big(F(X \sqcap Y) \sqsubseteq (FX \sqcap FY)\big)$$

$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcup Y)\big)$$

$$A_{\vec{\alpha}\,t} \sqcap B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \wedge B\vec{x} \qquad A_{\vec{\alpha}\,t} \sqcup B_{\vec{\alpha}\,t} \overset{\text{def}}{=} \lambda\vec{x}.A\vec{x} \vee B\vec{x}$$



$$\begin{array}{c|c} \multicolumn{2}{l}{\uparrow\sqsubseteq} \\ \hline \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array} \; + \; \begin{array}{c|c} \multicolumn{2}{l}{\uparrow\sqsubseteq} \\ \hline \multicolumn{2}{c}{G\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H^{\uparrow}\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [A, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [A, \vec{C}] : \mathbb{F} \end{array} \; = \; \begin{array}{c|c} \multicolumn{2}{l}{\uparrow\sqsubseteq} \\ \hline \multicolumn{2}{c}{G\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [P, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [P, \vec{C}] : \mathbb{F} \\ \hline \multicolumn{2}{c}{G^{\uparrow} \text{ and } P = B, \text{ or } H^{\uparrow} \text{ and } P = A} \end{array}$$

Monotonicity rules (Downward)

Definition (Downward monotonicity)

A function term F of type $(\vec{\alpha}\,t)\vec{\gamma}\,t$ is downward monotone ($\downarrow$), denoted as $F^{\downarrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \rightarrow (FY \sqsubseteq FX)\big)$$
$$\forall XY\big(F(X \sqcup Y) \sqsubseteq (FX \sqcap FY)\big)$$
$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcap Y)\big)$$

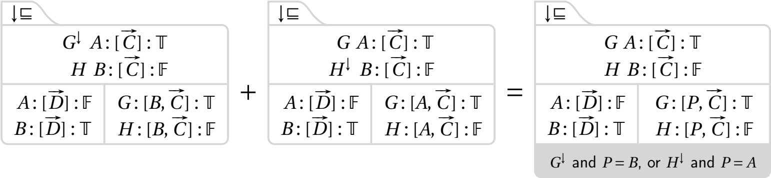
$$\begin{array}{c|c} \multicolumn{2}{l}{\downarrow\sqsubseteq} \\ \hline \multicolumn{2}{c}{G^{\downarrow}\ A:[\vec{C}]:\mathbb{T}} \\ \multicolumn{2}{c}{H\ B:[\vec{C}]:\mathbb{F}} \\ \hline A:[\vec{D}]:\mathbb{F} & G:[B,\vec{C}]:\mathbb{T} \\ B:[\vec{D}]:\mathbb{T} & H:[B,\vec{C}]:\mathbb{F} \end{array} \quad + \quad \begin{array}{c|c} \multicolumn{2}{l}{\downarrow\sqsubseteq} \\ \hline \multicolumn{2}{c}{G\ A:[\vec{C}]:\mathbb{T}} \\ \multicolumn{2}{c}{H^{\downarrow}\ B:[\vec{C}]:\mathbb{F}} \\ \hline A:[\vec{D}]:\mathbb{F} & G:[A,\vec{C}]:\mathbb{T} \\ B:[\vec{D}]:\mathbb{T} & H:[A,\vec{C}]:\mathbb{F} \end{array}$$

Monotonicity rules (Downward)

Definition (Downward monotonicity)

A function term F of type $(\vec{\alpha}\,t)\vec{\gamma}\,t$ is downward monotone ($\downarrow$), denoted as $F^{\downarrow}$, if it satisfies one of the following equivalent properties:

$$\forall XY\big((X \sqsubseteq Y) \to (FY \sqsubseteq FX)\big)$$
$$\forall XY\big(F(X \sqcup Y) \sqsubseteq (FX \sqcap FY)\big)$$
$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcap Y)\big)$$



$$\downarrow\sqsubseteq\ \dfrac{\begin{array}{c} G^{\downarrow}\ A:[\vec{C}]:\mathbb{T} \\ H\ B:[\vec{C}]:\mathbb{F} \end{array}}{\begin{array}{c|c} A:[\vec{D}]:\mathbb{F} & G:[B,\vec{C}]:\mathbb{T} \\ B:[\vec{D}]:\mathbb{T} & H:[B,\vec{C}]:\mathbb{F} \end{array}} \quad + \quad \downarrow\sqsubseteq\ \dfrac{\begin{array}{c} G\ A:[\vec{C}]:\mathbb{T} \\ H^{\downarrow}\ B:[\vec{C}]:\mathbb{F} \end{array}}{\begin{array}{c|c} A:[\vec{D}]:\mathbb{F} & G:[A,\vec{C}]:\mathbb{T} \\ B:[\vec{D}]:\mathbb{T} & H:[A,\vec{C}]:\mathbb{F} \end{array}} \quad = \quad \downarrow\sqsubseteq\ \dfrac{\begin{array}{c} G\ A:[\vec{C}]:\mathbb{T} \\ H\ B:[\vec{C}]:\mathbb{F} \end{array}}{\begin{array}{c|c} A:[\vec{D}]:\mathbb{F} & G:[P,\vec{C}]:\mathbb{T} \\ B:[\vec{D}]:\mathbb{T} & H:[P,\vec{C}]:\mathbb{F} \end{array}}$$

$G^{\downarrow}$ and $P = B$, or $H^{\downarrow}$ and $P = A$

Monotonicity rules in action

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\downarrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\subset$

$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

$$\forall X (A^{\subset} X \sqsubseteq X)$$

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Monotonicity rules in action

$\boxed{1}$ `every prover (quickly halt)` $: [\,] : \mathbb{T}$

$\boxed{2}$ `most (tableau prover) terminate` $: [\,] : \mathbb{F}$

↑⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}\,] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

↓⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}\,] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

⊂

$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

$$\forall X (A^{\subset} X \sqsubseteq X)$$

×⊑

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Monotonicity rules in action

$\boxed{1}$ $\texttt{every prover (quickly halt)} : [\,] : \mathbb{T}$

$\boxed{2}$ $\texttt{most (tableau prover) terminate} : [\,] : \mathbb{F}$

↑⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}\,] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ \ B : [\vec{C}\,] : \mathbb{F}} \\ \hline A : [\,\vec{d}\,] : \mathbb{T} & G : [B, \vec{C}\,] : \mathbb{T} \\ B : [\,\vec{d}\,] : \mathbb{F} & H : [B, \vec{C}\,] : \mathbb{F} \end{array}$$

↓⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}\,] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ \ B : [\vec{C}\,] : \mathbb{F}} \\ \hline A : [\,\vec{d}\,] : \mathbb{F} & G : [B, \vec{C}\,] : \mathbb{T} \\ B : [\,\vec{d}\,] : \mathbb{T} & H : [B, \vec{C}\,] : \mathbb{F} \end{array}$$

⊂

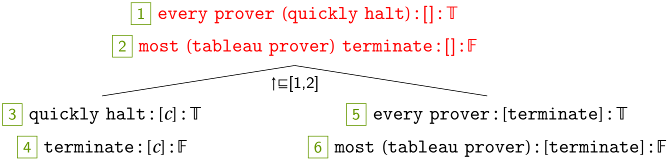
$$\begin{array}{c} A^{\subset}\ N : [\vec{C}\,] : \mathbb{T} \\ \hline N : [\vec{C}\,] : \mathbb{T} \end{array}$$

$\forall X (A^{\subset} X \sqsubseteq X)$

×⊑

$$\begin{array}{c} A : [\vec{C}\,] : \mathbb{T} \\ B : [\vec{C}\,] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Monotonicity rules in action



$\boxed{1}$ `every prover (quickly halt)` $: [\,] : \mathbb{T}$

$\boxed{2}$ `most (tableau prover) terminate` $: [\,] : \mathbb{F}$

$\uparrow\sqsubseteq$[1,2]

$\boxed{3}$ `quickly halt` $: [c] : \mathbb{T}$

$\boxed{4}$ `terminate` $: [c] : \mathbb{F}$

$\boxed{5}$ `every prover` $: [$`terminate`$] : \mathbb{T}$

$\boxed{6}$ `most (tableau prover)` $: [$`terminate`$] : \mathbb{F}$

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}\,] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\downarrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}\,] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\subset$

$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

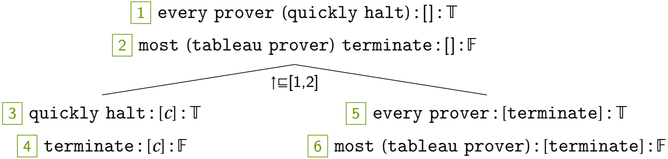
$\forall X(A^{\subset}X \sqsubseteq X)$

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \end{array}$$

$A \sqsubseteq B$

Monotonicity rules in action



1 `every prover (quickly halt)` $: [\,] : \mathbb{T}$

2 `most (tableau prover) terminate` $: [\,] : \mathbb{F}$

$\uparrow\sqsubseteq[1,2]$

3 `quickly halt` $: [c] : \mathbb{T}$

4 `terminate` $: [c] : \mathbb{F}$

5 `every prover` $:$ `[terminate]` $: \mathbb{T}$

6 `most (tableau prover)` $:$ `[terminate]` $: \mathbb{F}$

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\downarrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\subset$

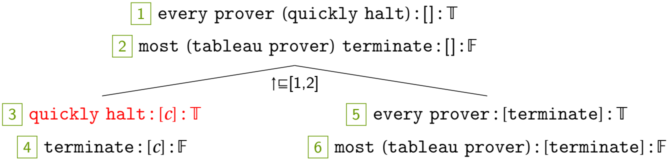
$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

$$\forall X (A^{\subset} X \sqsubseteq X)$$

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Monotonicity rules in action



$\boxed{1}$ every prover (quickly halt) $: [\,] : \mathbb{T}$

$\boxed{2}$ most (tableau prover) terminate $: [\,] : \mathbb{F}$

$\uparrow\sqsubseteq[1,2]$

$\boxed{3}$ quickly halt $: [c] : \mathbb{T}$

$\boxed{4}$ terminate $: [c] : \mathbb{F}$

$\boxed{5}$ every prover $:$ [terminate] $: \mathbb{T}$

$\boxed{6}$ most (tableau prover) $:$ [terminate] $: \mathbb{F}$

$\uparrow\sqsubseteq$

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}\,] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

$\downarrow\sqsubseteq$

$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}\,] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

$\subset$

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

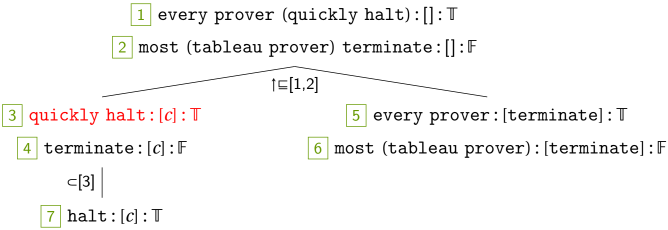
$\forall X (A^{\subset} X \sqsubseteq X)$

$\times\sqsubseteq$

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

$A \sqsubseteq B$

Monotonicity rules in action



1 `every prover (quickly halt)` $: [\,] : \mathbb{T}$

2 `most (tableau prover) terminate` $: [\,] : \mathbb{F}$

$\uparrow\sqsubseteq[1,2]$

3 `quickly halt` $: [c] : \mathbb{T}$

4 `terminate` $: [c] : \mathbb{F}$

$\subset[3]$

7 `halt` $: [c] : \mathbb{T}$

5 `every prover` $: [$`terminate`$] : \mathbb{T}$

6 `most (tableau prover)` $: [$`terminate`$] : \mathbb{F}$

$\uparrow\sqsubseteq$

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}\,] : \mathbb{T},\ B : [\vec{d}\,] : \mathbb{F} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T},\ H : [B, \vec{C}] : \mathbb{F}}$$

$\downarrow\sqsubseteq$

$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}\,] : \mathbb{F},\ B : [\vec{d}\,] : \mathbb{T} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T},\ H : [B, \vec{C}] : \mathbb{F}}$$

$\subset$

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

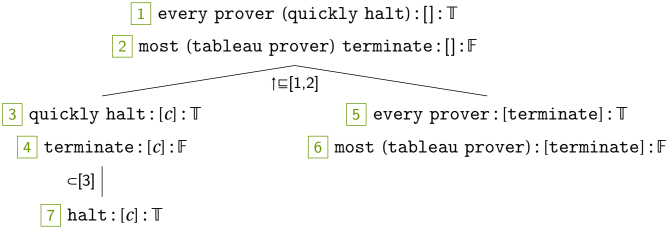
$$\forall X (A^{\subset} X \sqsubseteq X)$$

$\times\sqsubseteq$

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

$$A \sqsubseteq B$$

Monotonicity rules in action



1 `every prover (quickly halt)` : [] : $\mathbb{T}$

2 `most (tableau prover) terminate` : [] : $\mathbb{F}$

↑⊑[1,2]

3 `quickly halt` : [c] : $\mathbb{T}$

4 `terminate` : [c] : $\mathbb{F}$

⊂[3]

7 `halt` : [c] : $\mathbb{T}$

5 `every prover` : [`terminate`] : $\mathbb{T}$

6 `most (tableau prover)` : [`terminate`] : $\mathbb{F}$

↑⊑

$G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}$
$H\ B : [\vec{C}] : \mathbb{F}$

$A : [\vec{d}] : \mathbb{T}$ $B : [\vec{d}] : \mathbb{F}$	$G : [B, \vec{C}] : \mathbb{T}$ $H : [B, \vec{C}] : \mathbb{F}$

↓⊑

$G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}$
$H\ B : [\vec{C}] : \mathbb{F}$

$A : [\vec{d}] : \mathbb{F}$ $B : [\vec{d}] : \mathbb{T}$	$G : [B, \vec{C}] : \mathbb{T}$ $H : [B, \vec{C}] : \mathbb{F}$

⊂

$A^{\subset}\ N : [\vec{C}] : \mathbb{T}$

$N : [\vec{C}] : \mathbb{T}$

$\forall X (A^{\subset} X \sqsubseteq X)$

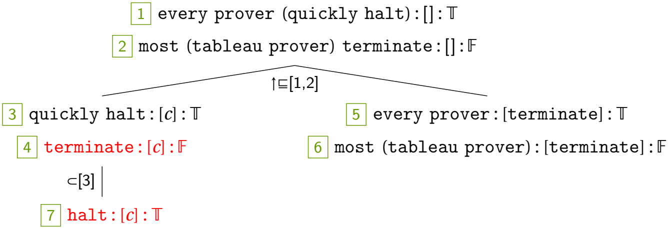
×⊑

$A : [\vec{C}] : \mathbb{T}$
$B : [\vec{C}] : \mathbb{F}$

×

$A \sqsubseteq B$

Monotonicity rules in action



1 every prover (quickly halt) : [] : $\mathbb{T}$

2 most (tableau prover) terminate : [] : $\mathbb{F}$

↑⊑[1,2]

3 quickly halt : [c] : $\mathbb{T}$

4 terminate : [c] : $\mathbb{F}$

⊂[3]

7 halt : [c] : $\mathbb{T}$

5 every prover : [terminate] : $\mathbb{T}$

6 most (tableau prover) : [terminate] : $\mathbb{F}$

↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}] : \mathbb{T}\quad B : [\vec{d}] : \mathbb{F} \;\Big|\; G : [B, \vec{C}] : \mathbb{T}\quad H : [B, \vec{C}] : \mathbb{F}}$$

↓⊑

$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}] : \mathbb{F}\quad B : [\vec{d}] : \mathbb{T} \;\Big|\; G : [B, \vec{C}] : \mathbb{T}\quad H : [B, \vec{C}] : \mathbb{F}}$$

⊂

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

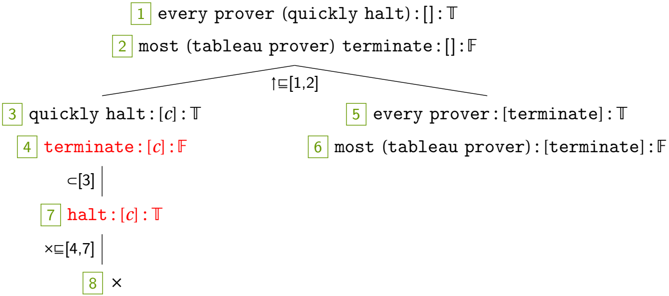
$\forall X(A^{\subset} X \sqsubseteq X)$

×⊑

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

$A \sqsubseteq B$

Monotonicity rules in action



1 every prover (quickly halt) : [] : $\mathbb{T}$

2 most (tableau prover) terminate : [] : $\mathbb{F}$

↑⊑[1,2]

3 quickly halt : $[c]$: $\mathbb{T}$

4 terminate : $[c]$: $\mathbb{F}$

⊂[3]

7 halt : $[c]$: $\mathbb{T}$

×⊑[4,7]

8 ×

5 every prover : [terminate] : $\mathbb{T}$

6 most (tableau prover) : [terminate] : $\mathbb{F}$

↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

↓⊑

$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

⊂

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

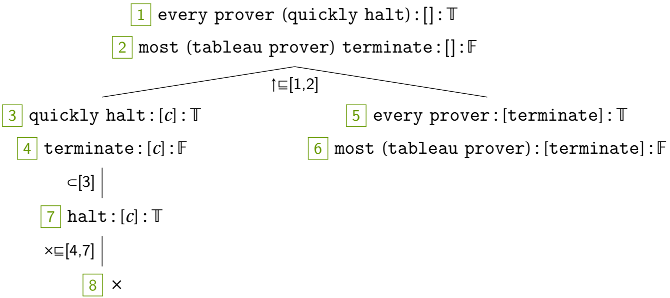
$\forall X (A^{\subset} X \sqsubseteq X)$

×⊑

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

$A \sqsubseteq B$

Monotonicity rules in action


1 every prover (quickly halt) : [] : 𝕋

2 most (tableau prover) terminate : [] : 𝔽

↑⊑[1,2]

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$$\frac{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}] : \mathbb{F},\ B : [\vec{d}] : \mathbb{T} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T},\ H : [B, \vec{C}] : \mathbb{F}}$$

⊂

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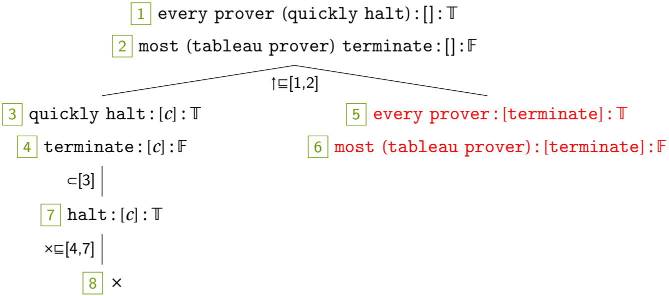
Monotonicity rules in action



$\boxed{1}$ every prover (quickly halt) : [] : $\mathbb{T}$

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$\uparrow\sqsubseteq$[1,2]

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$\boxed{4}$ terminate : $[c]$: $\mathbb{F}$

$\subset$[3]

$\boxed{7}$ halt : $[c]$: $\mathbb{T}$

$\times\sqsubseteq$[4,7]

$\boxed{8}$ $\times$

$\boxed{5}$ every prover : [terminate] : $\mathbb{T}$

$\boxed{6}$ most (tableau prover) : [terminate] : $\mathbb{F}$

$\uparrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\downarrow\sqsubseteq$

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

$\subset$

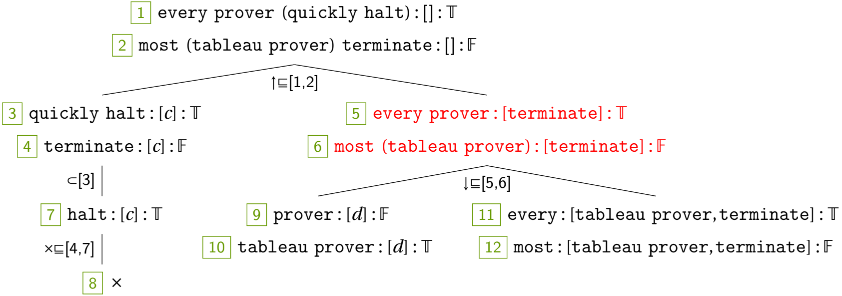
$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

$$\forall X (A^{\subset} X \sqsubseteq X)$$

$\times\sqsubseteq$

$$\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Monotonicity rules in action



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↓⊑[5,6]

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↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}\,] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

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⊂

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

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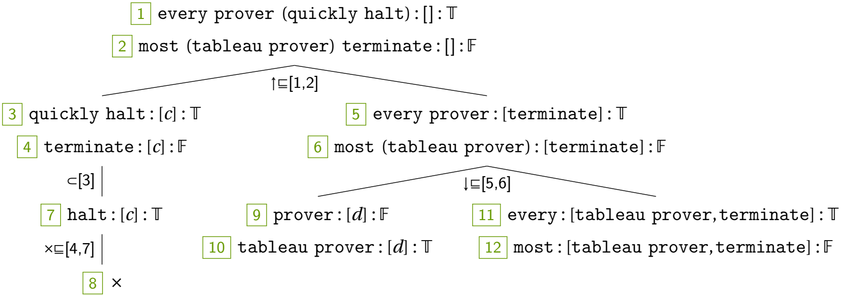
Monotonicity rules in action



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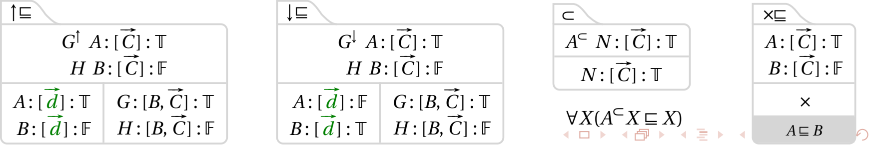

↑⊑

$$G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}$$

$$A : [\vec{d}] : \mathbb{T},\ B : [\vec{d}] : \mathbb{F} \quad \mid \quad G : [B, \vec{C}] : \mathbb{T},\ H : [B, \vec{C}] : \mathbb{F}$$

↓⊑

$$G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}$$

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⊂

$$A^{\subset}\ N : [\vec{C}] : \mathbb{T}$$

$$N : [\vec{C}] : \mathbb{T}$$

$$\forall X (A^{\subset} X \sqsubseteq X)$$

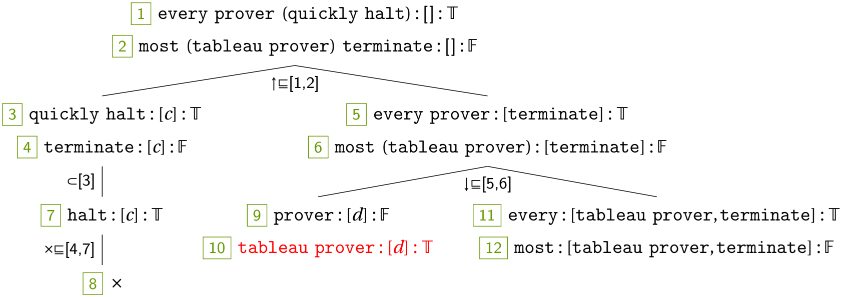
×⊑

$$A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}$$

$$\times$$

$$A \sqsubseteq B$$

Monotonicity rules in action



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↑⊑

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↓⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{d}\,] : \mathbb{F} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}\,] : \mathbb{T} & H : [B, \vec{C}] : \mathbb{F} \end{array}$$

⊂

$$\begin{array}{c} A^{\subset}\ N : [\vec{C}] : \mathbb{T} \\ \hline N : [\vec{C}] : \mathbb{T} \end{array}$$

$\forall X (A^{\subset} X \sqsubseteq X)$

×⊑

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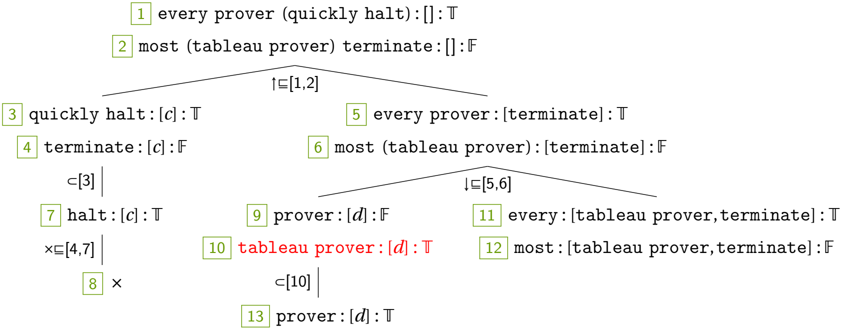
Monotonicity rules in action



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×⊑[4,7]

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$\boxed{9}$ prover : $[d]$: $\mathbb{F}$

$\boxed{10}$ tableau prover : $[d]$: $\mathbb{T}$

⊂[10]

$\boxed{13}$ prover : $[d]$: $\mathbb{T}$

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⊂

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×⊑

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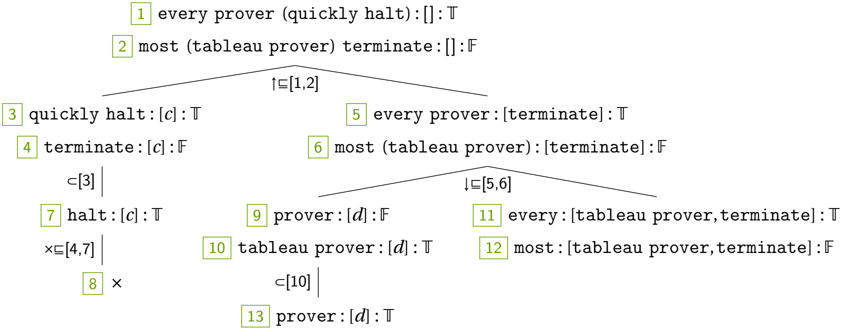
$A \sqsubseteq B$

Monotonicity rules in action



1 `every prover (quickly halt)` : $[\,]$: $\mathbb{T}$

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4 `terminate` : $[c]$: $\mathbb{F}$

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5 `every prover` : [`terminate`] : $\mathbb{T}$

6 `most (tableau prover)` : [`terminate`] : $\mathbb{F}$

$\downarrow\sqsubseteq[5,6]$

9 `prover` : $[d]$: $\mathbb{F}$

10 `tableau prover` : $[d]$: $\mathbb{T}$

$\subset[10]$

13 `prover` : $[d]$: $\mathbb{T}$

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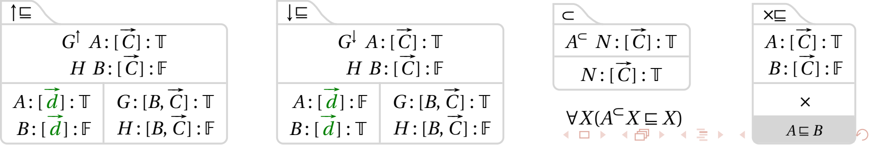
12 `most` : [`tableau prover`, `terminate`] : $\mathbb{F}$



$\uparrow\sqsubseteq$

$$\frac{G^{\uparrow}\ A:[\vec{C}]:\mathbb{T} \qquad H\ B:[\vec{C}]:\mathbb{F}}{A:[\vec{d}\,]:\mathbb{T} \quad B:[\vec{d}\,]:\mathbb{F} \;\Big|\; G:[B,\vec{C}]:\mathbb{T} \quad H:[B,\vec{C}]:\mathbb{F}}$$

$\downarrow\sqsubseteq$

$$\frac{G^{\downarrow}\ A:[\vec{C}]:\mathbb{T} \qquad H\ B:[\vec{C}]:\mathbb{F}}{A:[\vec{d}\,]:\mathbb{F} \quad B:[\vec{d}\,]:\mathbb{T} \;\Big|\; G:[B,\vec{C}]:\mathbb{T} \quad H:[B,\vec{C}]:\mathbb{F}}$$

$\subset$

$$\frac{A^{\subset}\ N:[\vec{C}]:\mathbb{T}}{N:[\vec{C}]:\mathbb{T}}$$

$\forall X(A^{\subset}X \sqsubseteq X)$

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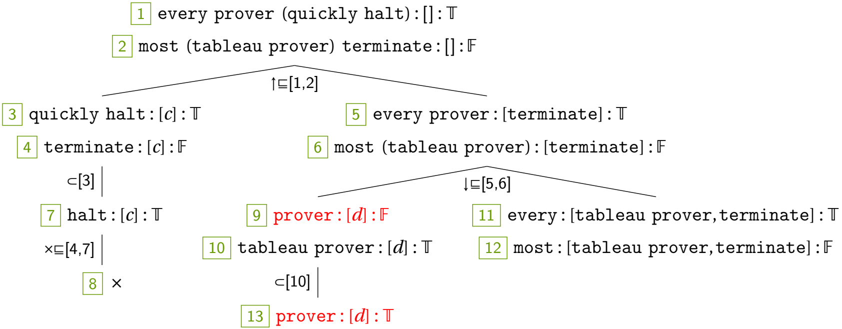
Monotonicity rules in action



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⊂[10]

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↑⊑

$G^{\uparrow}\ A : [\vec{C}] : \mathbb{T}$

$H\ B : [\vec{C}] : \mathbb{F}$

$A : [\vec{d}] : \mathbb{T}$ $B : [\vec{d}] : \mathbb{F}$	$G : [B, \vec{C}] : \mathbb{T}$ $H : [B, \vec{C}] : \mathbb{F}$

↓⊑

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⊂

$A^{\subset}\ N : [\vec{C}] : \mathbb{T}$

$N : [\vec{C}] : \mathbb{T}$

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×⊑

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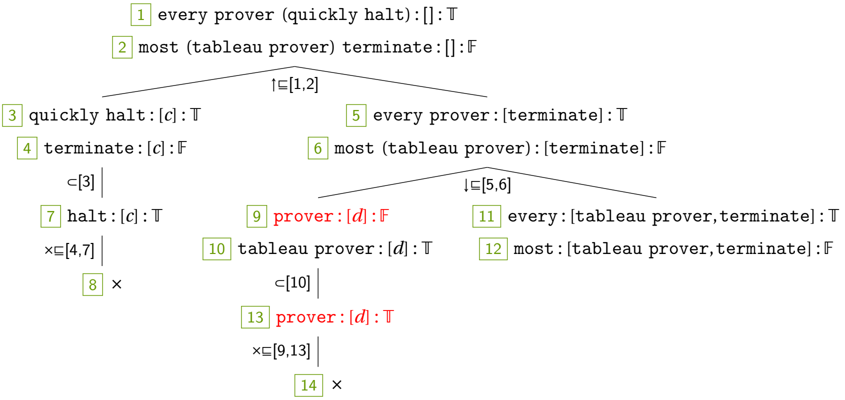
×

$A \sqsubseteq B$

Monotonicity rules in action



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⊂[10]
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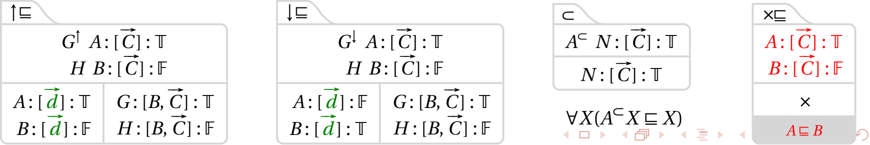
×⊑[9,13]
14 ×

11 every : [tableau prover, terminate] : $\mathbb{T}$
12 most : [tableau prover, terminate] : $\mathbb{F}$



↑⊑
$G^{\uparrow}\ A:[\vec{C}]:\mathbb{T}$
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↓⊑
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⊂
$A^{\subset}\ N:[\vec{C}]:\mathbb{T}$
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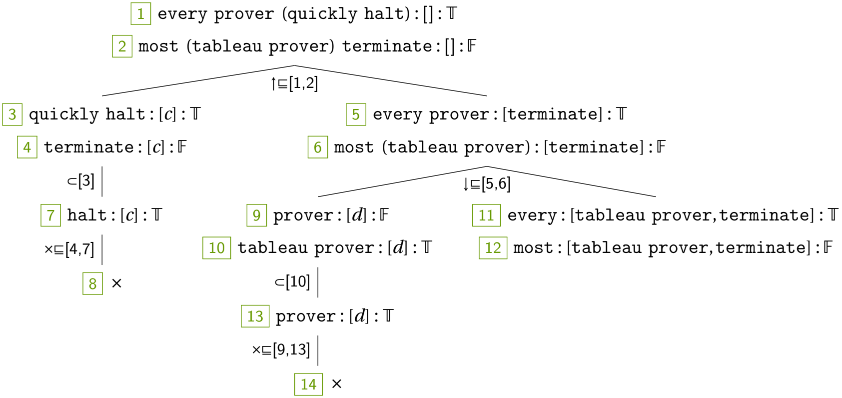
$\forall X(A^{\subset}X \sqsubseteq X)$

×⊑
$A:[\vec{C}]:\mathbb{T}$
$B:[\vec{C}]:\mathbb{F}$
×
$A \sqsubseteq B$

Monotonicity rules in action



↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{A : [\vec{d}] : \mathbb{T},\ B : [\vec{d}] : \mathbb{F} \quad \Big| \quad G : [B, \vec{C}] : \mathbb{T},\ H : [B, \vec{C}] : \mathbb{F}}$$

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⊂

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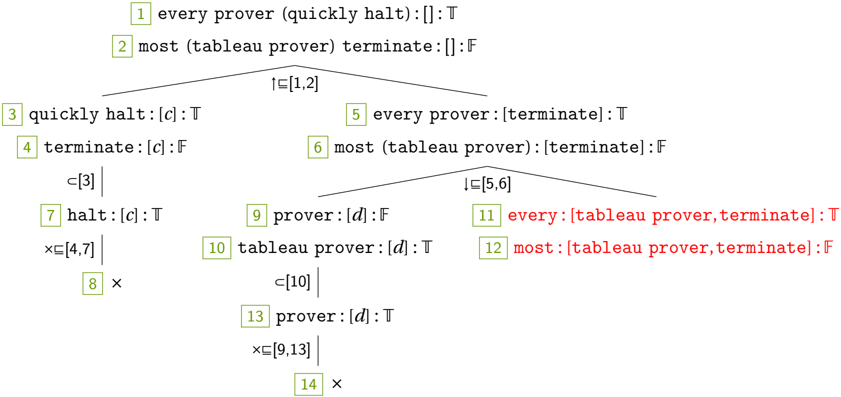
Monotonicity rules in action



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10 `tableau prover` : $[d]$: $\mathbb{T}$

⊂[10]

13 `prover` : $[d]$: $\mathbb{T}$

×⊑[9,13]

14 ×

11 `every` : [`tableau prover`, `terminate`] : $\mathbb{T}$

12 `most` : [`tableau prover`, `terminate`] : $\mathbb{F}$

↑⊑

$$\frac{\begin{array}{c} G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \\ H\ B : [\vec{C}] : \mathbb{F} \end{array}}{\begin{array}{c} A : [\vec{d}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} \end{array} \;\Big|\; \begin{array}{c} G : [B, \vec{C}] : \mathbb{T} \\ H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

↓⊑

$$\frac{\begin{array}{c} G^{\downarrow}\ A : [\vec{C}] : \mathbb{T} \\ H\ B : [\vec{C}] : \mathbb{F} \end{array}}{\begin{array}{c} A : [\vec{d}] : \mathbb{F} \\ B : [\vec{d}] : \mathbb{T} \end{array} \;\Big|\; \begin{array}{c} G : [B, \vec{C}] : \mathbb{T} \\ H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

⊂

$$\frac{A^{\subset}\ N : [\vec{C}] : \mathbb{T}}{N : [\vec{C}] : \mathbb{T}}$$

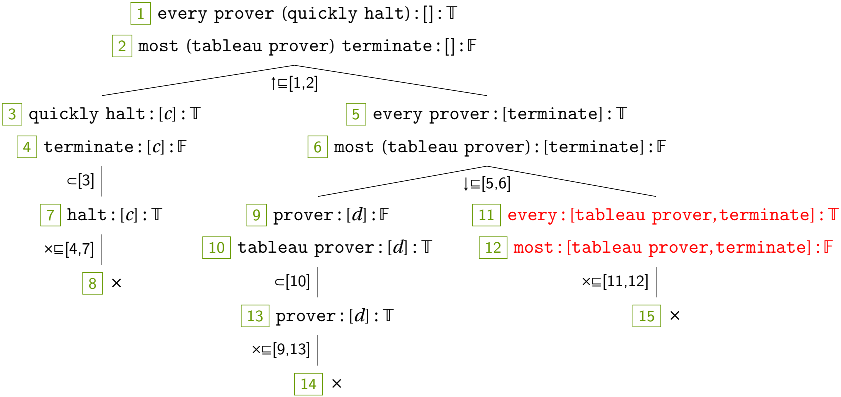
$$\forall X (A^{\subset} X \sqsubseteq X)$$

×⊑

$$\frac{\begin{array}{c} A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \end{array}}{\times}$$

$A \sqsubseteq B$

Monotonicity rules in action



1 every prover (quickly halt) : [] : $\mathbb{T}$

2 most (tableau prover) terminate : [] : $\mathbb{F}$

↑⊑[1,2]

3 quickly halt : $[c]$: $\mathbb{T}$

4 terminate : $[c]$: $\mathbb{F}$

⊂[3]

7 halt : $[c]$: $\mathbb{T}$

×⊑[4,7]

8 ×

5 every prover : [terminate] : $\mathbb{T}$

6 most (tableau prover) : [terminate] : $\mathbb{F}$

↓⊑[5,6]

9 prover : $[d]$: $\mathbb{F}$

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⊂[10]

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11 every : [tableau prover, terminate] : $\mathbb{T}$

12 most : [tableau prover, terminate] : $\mathbb{F}$

×⊑[11,12]

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↑⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\uparrow}\ A:[\vec{C}]:\mathbb{T}} \\ \multicolumn{2}{c}{H\ B:[\vec{C}]:\mathbb{F}} \\ \hline A:[\vec{d}\,]:\mathbb{T} & G:[B,\vec{C}]:\mathbb{T} \\ B:[\vec{d}\,]:\mathbb{F} & H:[B,\vec{C}]:\mathbb{F} \end{array}$$

↓⊑

$$\begin{array}{c|c} \multicolumn{2}{c}{G^{\downarrow}\ A:[\vec{C}]:\mathbb{T}} \\ \multicolumn{2}{c}{H\ B:[\vec{C}]:\mathbb{F}} \\ \hline A:[\vec{d}\,]:\mathbb{F} & G:[B,\vec{C}]:\mathbb{T} \\ B:[\vec{d}\,]:\mathbb{T} & H:[B,\vec{C}]:\mathbb{F} \end{array}$$

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×⊑

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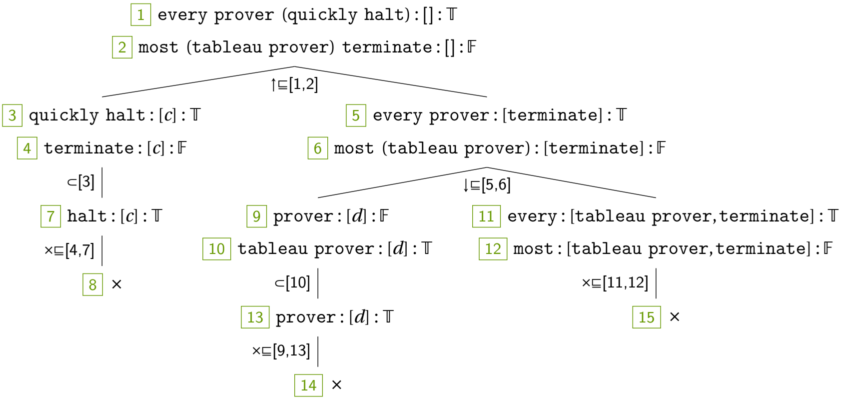
Monotonicity rules in action



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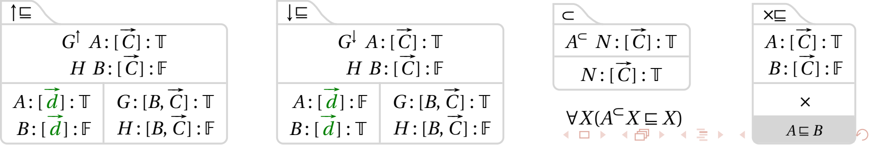

↑⊑

$$\frac{G^{\uparrow}\ A : [\vec{C}] : \mathbb{T} \qquad H\ B : [\vec{C}] : \mathbb{F}}{\begin{array}{c|c} A : [\vec{d}] : \mathbb{T} & G : [B, \vec{C}] : \mathbb{T} \\ B : [\vec{d}] : \mathbb{F} & H : [B, \vec{C}] : \mathbb{F} \end{array}}$$

↓⊑

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×⊑

$$\frac{A : [\vec{C}] : \mathbb{T} \qquad B : [\vec{C}] : \mathbb{F}}{\times}$$

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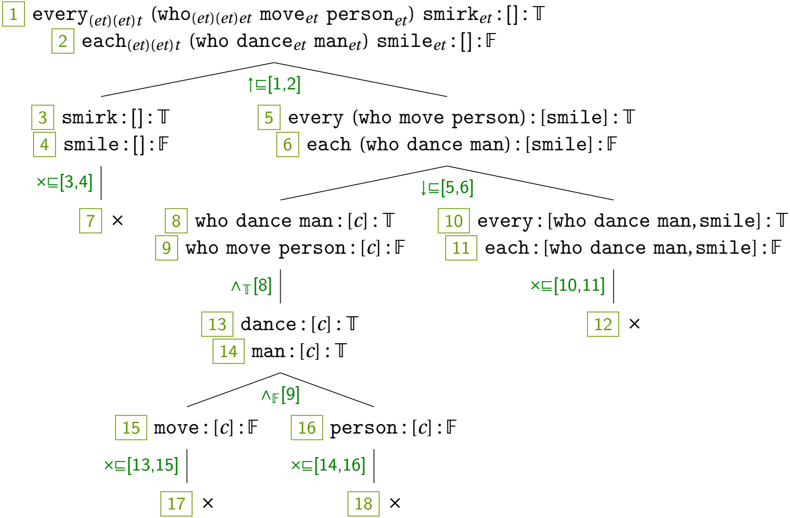
Monotonicity rules in action (II)



1 $\texttt{every}_{(et)(et)t}$ ($\texttt{who}_{(et)(et)et}$ $\texttt{move}_{et}$ $\texttt{person}_{et}$) $\texttt{smirk}_{et}$: [] : $\mathbb{T}$

2 $\texttt{each}_{(et)(et)t}$ ($\texttt{who}$ $\texttt{dance}_{et}$ $\texttt{man}_{et}$) $\texttt{smile}_{et}$: [] : $\mathbb{F}$

3 smirk : [] : $\mathbb{T}$

4 smile : [] : $\mathbb{F}$

7 ×

5 every (who move person) : [smile] : $\mathbb{T}$

6 each (who dance man) : [smile] : $\mathbb{F}$

8 who dance man : [c] : $\mathbb{T}$

9 who move person : [c] : $\mathbb{F}$

13 dance : [c] : $\mathbb{T}$

14 man : [c] : $\mathbb{T}$

15 move : [c] : $\mathbb{F}$

17 ×

16 person : [c] : $\mathbb{F}$

18 ×

10 every : [who dance man, smile] : $\mathbb{T}$

11 each : [who dance man, smile] : $\mathbb{F}$

12 ×

Rules getting rid of Boolean connectives

Remember other two equivalent conditions for upward monotonicity:

$$\forall XY\big((X \sqsubseteq Y) \to (FX \sqsubseteq FY)\big)$$

$$\forall XY\big(F(X \sqcap Y) \sqsubseteq (FX \sqcap FY)\big)$$

$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcup Y)\big)$$

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$$\forall XY\big((X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)\big)$$

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$$\forall XY\big((FX \sqcup FY) \sqsubseteq F(X \sqcup Y)\big)$$

These conditions give rise to:

F$\sqcap_{\mathbb{T}}$

$$\frac{F^{\uparrow}(\text{and } A\ B) : [\vec{C}] : \mathbb{T}}{\begin{array}{c} FA : [\vec{C}] : \mathbb{T} \\ FB : [\vec{C}] : \mathbb{T} \end{array}}$$

$\boxed{1}$ every dog (and run bark) : [] : $\mathbb{T}$

F$\sqcap_{\mathbb{T}}$[1]

$\boxed{2}$ every dog run : [] : $\mathbb{T}$

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F$\sqcup_{\mathbb{F}}$

$$\frac{F^{\uparrow}(\text{or } A\ B) : [\vec{C}] : \mathbb{F}}{\begin{array}{c} FA : [\vec{C}] : \mathbb{F} \\ FB : [\vec{C}] : \mathbb{F} \end{array}}$$

1 `every dog (and run bark)` $: [\,] : \mathbb{T}$

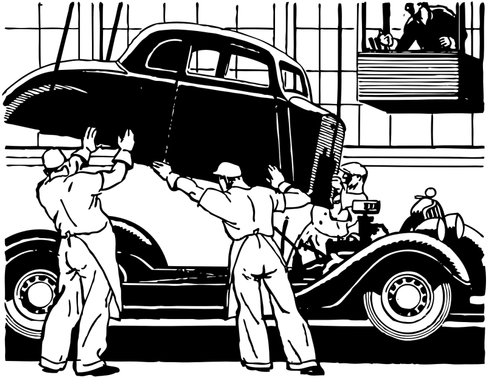
F$\sqcap_{\mathbb{T}}$[1]

2 `every dog run` $: [\,] : \mathbb{T}$
3 `every dog bark` $: [\,] : \mathbb{T}$

1 `most cat (or meow sleep)` $: [\,] : \mathbb{F}$

F$\sqcup_{\mathbb{F}}$[1]

2 `most cat meow` $: [\,] : \mathbb{F}$
3 `most cat sleep` $: [\,] : \mathbb{F}$



Tuning Natural Tableau

Uninformative *et*-based types

Semantic types based on e and t are uninformative from a syntactic point of view:

$$h_{et} : [c_e] \qquad \begin{array}{l} \mathtt{cat}_{et} : [c_e] \\ \quad \text{or} \\ \mathtt{sleep}_{et} : [c_e] \end{array}$$

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$$A_{(et)et}\ B_{et} : [c_e] \qquad \begin{array}{l} \mathtt{little}_{(et)et}\ \mathtt{bird}_{et} : [c_e] \\ \text{or} \\ \mathtt{high}_{(et)et}\ \mathtt{fly}_{et} : [c_e] \end{array}$$

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$\texttt{little}_{(et)et}\ \texttt{bird}_{et} : [c_e]$
or
$\texttt{high}_{(et)et}\ \texttt{fly}_{et} : [c_e]$

$a_{(et)et}(b_{eet} c_e) : [c_e]$

$\texttt{quietly}_{(et)et}\ (\texttt{follow}_{eet}\ \texttt{john}_e) : [c_e]$
or
$\texttt{wife}_{(et)et}\ (\texttt{of}_{eet}\ \texttt{john}_e) : [c_e]$

Extending the type system

Add syntactic types to semantic ones:

$$\{e, t\} + \{\mathrm{np}, \mathrm{s}, \mathrm{n}, \mathrm{pp}\}$$

A partial order subtyping relation (<:) serves as an interface between syntactic and semantic types:

- $\mathrm{s} <: t$
- $e <: \mathrm{np}$
- $\mathrm{n} <: et$
- $\mathrm{pp} <: et$
- $(\alpha_1, \alpha_2) <: (\beta_1, \beta_2)$ iff $\beta_1 <: \alpha_1$ and $\alpha_2 <: \beta_2$

Syntactic terms

An additional typing rule:

if $A : \alpha$ and $\alpha <: \beta$, then $A : \beta$ too.

Terms of multiple types:

- **cat**$_{\mathrm{n}}$ is of type et
- **red**$_{\mathrm{n,n}}$ is of type (n, et) ~~and (et, et)~~
- **see**$_{\mathrm{np,np,s}}$ is of type $\mathrm{np}(\mathrm{np}, t)$, eet, ...

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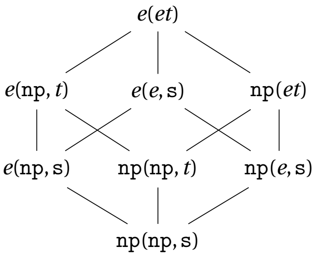
Types of $\mathbf{see}_{\mathrm{np,np,s}}$



$e(et)$

$e(\mathrm{np}, t)$ $e(e, \mathrm{s})$ $\mathrm{np}(et)$

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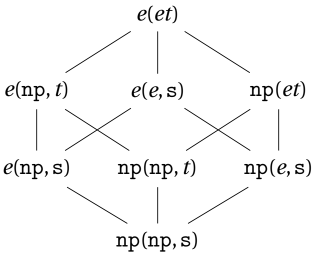
Syntactic and semantic terms together:

cat$_{\mathrm{n}}\, c_e$, **love**$_{\mathrm{np,np,s}}$**john**$_{\mathrm{np}}\, c_e$, **on**$_{\mathrm{pp}}\, d_e$

Types of **see**$_{\mathrm{np,np,s}}$



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$e(\mathrm{np}, t)$ $e(e, \mathrm{s})$ $\mathrm{np}(et)$

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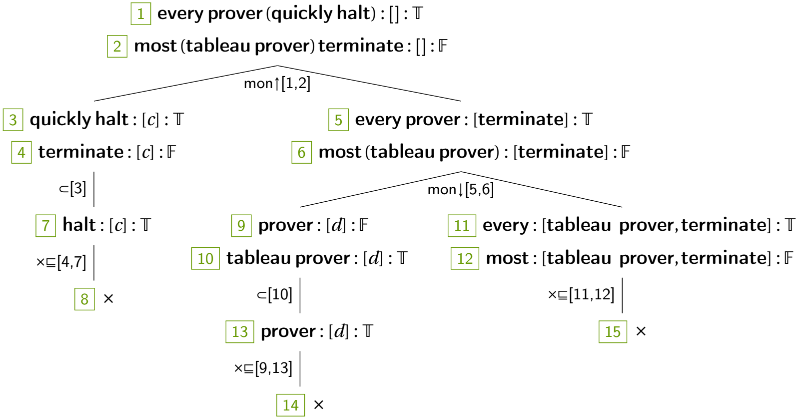
$\mathrm{np}(\mathrm{np}, \mathrm{s})$

No much changes in tableau proofs

Only the **style** of the terms is changed



1 **every prover (quickly halt)** : [] : $\mathbb{T}$

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mon↑[1,2]

3 **quickly halt** : $[c]$: $\mathbb{T}$

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⊂[3]

7 **halt** : $[c]$: $\mathbb{T}$

×⊑[4,7]

8 ×

5 **every prover** : [**terminate**] : $\mathbb{T}$

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mon↓[5,6]

9 **prover** : $[d]$: $\mathbb{F}$

10 **tableau prover** : $[d]$: $\mathbb{T}$

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×⊑[11,12]

15 ×

Remote modifiers

A syntactic head can be modified by several modifiers:

- Adverbs and prepositional phrases @ a verb
  John jogged slowly in Riga at midnight
- Adjective and PPs @ a noun
  small old yellow Soviet bus

src: http://psnews.ge/index.php?m=68&news_id=6229

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Lexicalized compositional approaches:

- [Winter and Zwarts, 2011]:
  $\exists\!-\!CLOS_{V,S}\ \big(ATMIDNIGHT_{V,V}\ (INRIGA_{V,V}\ (SLOWLY_{V,V}\ (JOG_{NP,V}\ JOHN_{NP})))\big)$

The tableau method unfolds from top to bottom
Lexicalized compositional methods from bottom to top

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- [Champollion, 2014]:

⟦CLOSURE⟧ (⟦AG⟧ ⟦john⟧ (⟦slowly⟧ (⟦LOC⟧ ⟦Riga⟧ (⟦TIM⟧ ⟦midnight⟧ ⟦jog⟧))))

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How to maintain natural appearance of LLFs while accounting for remote modifiers?

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small old yellow Soviet bus

$$\textbf{small}_{\mathrm{n,n}}\Big(\textbf{old}_{\mathrm{n,n}}\big(\textbf{yellow}_{\mathrm{n,n}}(\textbf{soviet}_{\mathrm{n,n}}\textbf{bus}_{\mathrm{n}})\big)\Big)$$

Function list

$$\underbrace{\mathtt{FuncList : LLF : argumentList}}_{\text{ternary format of a term}} : \mathtt{truthSign}$$

Correspondence to the syntactic trichotomy:

$$\mathtt{Adjuncts : Head : Complements}$$

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Correspondence to the syntactic trichotomy:

`Adjuncts:Head:Complements`

New tableau rules for the function list:

M<

$$[\overrightarrow{M}] : A\ H : [\overrightarrow{C}] : \mathbb{X}$$

$$[\overrightarrow{M}, A] : H : [\overrightarrow{C}] : \mathbb{X}$$

M>

$$[\overrightarrow{M}, A] : H : [\overrightarrow{C}] : \mathbb{X}$$

$$[\overrightarrow{M}] : A\ H : [\overrightarrow{C}] : \mathbb{X}$$

NB: **The term is uniquely recoverable from any of its ternary format**

Rule for events

EV$_{\mathbb{T}}$

$$[M^1_{\mathtt{vp,vp}}, \ldots, M^j_{\mathtt{vp,vp}}] : b_{\mathtt{np}^k,\mathtt{s}} : [c^1, \ldots, c^k] : \mathbb{T}$$

$$[A^1_{\mathtt{n,n}}, \ldots, A^j_{\mathtt{n,n}}] : b_{\mathtt{n}} : [v_e] : \mathbb{T}$$

$$\mathtt{role}^1_{eet} : [v, c^1_e] : \mathbb{T}$$

$$\vdots$$

$$\mathtt{role}^k_{eet} : [v, c^k_e] : \mathbb{T}$$

The verb b has the argument structure $[\mathtt{role}^1, \ldots, \mathtt{role}^l]$,
M^i is of type $\mathtt{vp,vp}$ and of form $p_{\mathtt{np,vp,vp}}H$ or lexical,
and $A^i_{\mathtt{n,n}} = p_{\mathtt{np,n,n}}H$ or $der(\mathtt{JJ}, M^i_{\mathtt{n,n}})$, respectively

Rule for events

$\mathrm{EV}_{\mathbb{T}}$

$$\frac{[M^1_{\mathrm{vp,vp}},\dots,M^j_{\mathrm{vp,vp}}] : b_{\mathrm{np}^k,\mathrm{s}} : [c^1,\dots,c^k] : \mathbb{T}}{\begin{array}{c}[A^1_{\mathrm{n,n}},\dots,A^j_{\mathrm{n,n}}] : b_{\mathrm{n}} : [v_e] : \mathbb{T} \\ \mathtt{role}^1_{eet} : [v, c^1_e] : \mathbb{T} \\ \vdots \\ \mathtt{role}^k_{eet} : [v, c^k_e] : \mathbb{T}\end{array}}$$

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$\boxed{1}$ $\mathsf{at\_midnight}_{\mathrm{vp,vp}}(\mathsf{in}_{\mathrm{np,vp,vp}}\mathsf{Riga}_{\mathrm{np}}(\mathsf{slowly}_{\mathrm{vp,vp}}\mathsf{jog}_{\mathrm{vp}}))\mathsf{John}_{\mathrm{np}} : [\,] : \mathbb{T}$

A>[1]

$\boxed{2}$ $\mathsf{at\_midnight}\ (\mathsf{in\ Riga}\ (\mathsf{slowly\ jog})) : [\mathsf{John}] : \mathbb{T}$

M<[2]

$\boxed{3}$ $[\mathsf{at\_midnight}] : \mathsf{in\ Riga}\ (\mathsf{slowly\ jog}) : [\mathsf{John}] : \mathbb{T}$

M<[3]

$\boxed{4}$ $[\mathsf{at\_midnight}, \mathsf{in\ Riga}] : \mathsf{slowly\ jog} : [\mathsf{John}] : \mathbb{T}$

M<[4]

$\boxed{5}$ $[\mathsf{at\_midnight}, \mathsf{in\ Riga}, \mathsf{slowly}] : \mathsf{jog} : [\mathsf{John}] : \mathbb{T}$

$\mathrm{EV}_{\mathbb{T}}$[5]

$\boxed{6}$ $[\mathsf{at\_midnight}_{\mathrm{n,n}}, \mathsf{in}_{\mathrm{np,n,n}}\mathsf{Riga}_{\mathrm{np}}, \mathsf{slow}_{\mathrm{n,n}}] : \mathsf{jog}_{\mathrm{n}} : [v_e] : \mathbb{T}$

$\boxed{7}$ $\mathtt{agent}_{eet} : [v_e, \mathtt{John}_e] : \mathbb{T}$

Rule for events

$\mathrm{EV}_{\mathbb{T}}$

$$[M^1_{\mathrm{vp,vp}}, \ldots, M^j_{\mathrm{vp,vp}}] : b_{\mathrm{np}^k,\mathrm{s}} : [c^1, \ldots, c^k] : \mathbb{T}$$

$$[A^1_{\mathrm{n,n}}, \ldots, A^j_{\mathrm{n,n}}] : b_{\mathrm{n}} : [v_e] : \mathbb{T}$$

$$\texttt{role}^1_{eet} : [v, c^1_e] : \mathbb{T}$$

$$\vdots$$

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The verb b has the argument structure $[\texttt{role}^1, \ldots, \texttt{role}^l]$, M^i is of type $\mathrm{vp,vp}$ and of form $p_{\mathrm{np,vp,vp}}H$ or lexical, and $A^i_{\mathrm{n,n}} = p_{\mathrm{np,n,n}}H$ or $der(\texttt{JJ}, M^i_{\mathrm{n,n}})$, respectively

$\boxed{1}$ $\mathsf{no}_{\mathrm{n,vp,s}}\ \mathsf{person}_{\mathrm{n}}\,(\mathsf{passionately}_{\mathrm{vp,vp}}\,(\mathsf{kiss}_{\mathrm{np,vp}}\ \mathsf{Mary}_{\mathrm{np}})) : [\,] : \mathbb{F}$

$\mathrm{NO}_{\mathbb{F}}[1]$

$\boxed{2}$ $\mathsf{person} : [c_e] : \mathbb{T}$

$\boxed{3}$ $\mathsf{passionately}\ (\mathsf{kiss\ Mary}) : [c_e] : \mathbb{T}$

$\mathrm{M}{<}[3]$

$\boxed{4}$ $[\mathsf{passionately}] : \mathsf{kiss\ Mary} : [c] : \mathbb{T}$

$\mathrm{A}{>}[4]$

$\boxed{5}$ $[\mathsf{passionately}] : \mathsf{kiss} : [\mathsf{Mary}, c] : \mathbb{T}$

$\mathrm{EV}_{\mathbb{T}}[5]$

$\boxed{6}$ $[\mathsf{passionate}_{\mathrm{n,n}}] : \mathsf{kiss}_{\mathrm{n}} : [v_e] : \mathbb{T}$

$\boxed{7}$ $\texttt{agent}_{eet} : [v, c] : \mathbb{T}$

$\boxed{8}$ $\texttt{theme}_{eet} : [v, \texttt{Mary}_e] : \mathbb{T}$

Rules for modifiers

M<

$[\vec{M}] : A\ H : [\vec{C}] : \mathbb{X}$

$[\vec{M}, A] : H : [\vec{C}] : \mathbb{X}$

$M^{\subset}$ when $\forall X(MX \sqsubseteq X)$

$\subset_{\mathbb{T}}$

$[\vec{M}] : A^{\subset}_{\alpha,\alpha} H : [\vec{C}] : \mathbb{T}$

$[\vec{M}] : H : [\vec{C}] : \mathbb{T}$

$\vec{M}^{\uparrow}$

$A \rightleftharpoons B \overset{\text{def}}{=} \forall X(ABX = BAX)$

$\rightleftharpoons\text{M}$

$[\vec{A}, M, M', \vec{B}] : H : [\vec{C}] : \mathbb{X}$

$[\vec{A}, M', M, \vec{B}] : H : [\vec{C}] : \mathbb{X}$

$M \rightleftharpoons M'$

$\subset\text{M}_{\mathbb{T}}$

$[M^{\subset}, \vec{A}] : H : [\vec{C}] : \mathbb{T}$

$[\vec{A}] : H : [\vec{C}] : \mathbb{T}$

$$\textbf{small}_{n,n}\Big(\textbf{old}_{n,n}\big(\textbf{yellow}_{n,n}(\textbf{soviet}_{n,n}\textbf{bus}_{n})\big)\Big) \vDash \textbf{old}_{n,n}(\textbf{soviet}_{n,n}\textbf{bus}_{n})$$

$\boxed{1}$ $\textbf{small}_{n,n}\Big(\textbf{old}_{n,n}\big(\textbf{yellow}_{n,n}(\textbf{soviet}_{n,n}\textbf{bus}_{n})\big)\Big) : \mathbb{T}$

$\boxed{2}$ $\textbf{old}_{n,n}(\textbf{soviet}_{n,n}\textbf{bus}_{n}) : \mathbb{F}$

$\subset_{\mathbb{T}}[1]$

$\boxed{3}$ $\textbf{old}_{n,n}\big(\textbf{yellow}_{n,n}(\textbf{soviet}_{n,n}\textbf{bus}_{n})\big) : \mathbb{T}$

$\text{M<,M<}[3]$

$\boxed{4}$ $[\textbf{old}_{n,n},\ \textbf{yellow}_{n,n}] : \textbf{soviet}_{n,n}\textbf{bus}_{n} : \mathbb{T}$

$\rightleftharpoons\text{M}[4]$

$\boxed{5}$ $[\textbf{yellow}_{n,n},\ \textbf{old}_{n,n}] : \textbf{soviet}_{n,n}\textbf{bus}_{n} : \mathbb{T}$

$\subset\text{M}_{\mathbb{T}}[5]$

$\boxed{6}$ $[\textbf{old}_{n,n}] : \textbf{soviet}_{n,n}\textbf{bus}_{n} : \mathbb{T}$

$\text{M<}[2]$

$\boxed{7}$ $[\textbf{old}_{n,n}] : \textbf{soviet}_{n,n}\textbf{bus}_{n} : \mathbb{F}$

$\times\sqsubseteq[6,7]$

$\boxed{8}$ $\times$

Rules for semantic exclusion and exhaustion

Definition (Exclusion)

Relational terms $A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are in an *exclusion* relation iff the following formula is true:

$$A \mid B \overset{\text{def}}{=} \neg\exists\vec{X}.\,(A \sqcap B)\,\vec{X}$$

×|

$a : [\vec{C}] : \mathbb{T}$
$b : [\vec{C}] : \mathbb{T}$

×

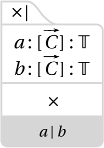
$a \mid b$

Rules for semantic exclusion and exhaustion

Definition (Exclusion)

Relational terms $A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are in an *exclusion* relation iff the following formula is true:

$$A \,|\, B \overset{\text{def}}{=} \neg\exists\vec{X}.(A \sqcap B)\vec{X}$$



$\times|$

$a : [\vec{C}] : \mathbb{T}$

$b : [\vec{C}] : \mathbb{T}$

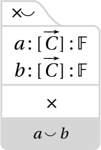
$\times$

$a \,|\, b$

Definition (Joint exhaustion)

Relational terms $A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are *jointly exhaustive* iff the following formula is true:

$$A \smile B \overset{\text{def}}{=} \forall\vec{X}.(A \sqcup B)\vec{X}$$



$\times\smile$

$a : [\vec{C}] : \mathbb{F}$

$b : [\vec{C}] : \mathbb{F}$

$\times$

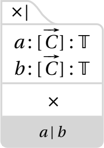
$a \smile b$

Rules for semantic exclusion and exhaustion

Definition (Exclusion)

Relational terms $A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are in an *exclusion* relation iff the following formula is true:

$$A \mid B \overset{\text{def}}{=} \neg\exists\vec{X}.(A \sqcap B)\vec{X}$$



×|

$a:[\vec{C}]:\mathbb{T}$

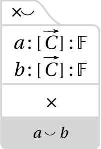
$b:[\vec{C}]:\mathbb{T}$

×

$a \mid b$

Definition (Joint exhaustion)

Relational terms $A_{\vec{\alpha}t}$ and $B_{\vec{\alpha}t}$ are *jointly exhaustive* iff the following formula is true: $A \smile B \overset{\text{def}}{=} \forall\vec{X}.(A \sqcup B)\vec{X}$



×⌣

$a:[\vec{C}]:\mathbb{F}$

$b:[\vec{C}]:\mathbb{F}$

×

$a \smile b$

(**dog** | **cat**), (**many** | **few**), (**sleep** | **run**)

(**nonhuman** ⌣ **animal**), (**at least six** ⌣ **at most ten**)

In both relations: ⟨**some**, **no**⟩, ⟨**with**, **without**⟩

Rules for semantic exclusion and exhaustion (II)

⇅

$$GA : [\vec{C}] : \mathbb{X}$$
$$HB : [\vec{C}] : \mathbb{X}$$

$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$	$G : [P, \vec{C}] : \mathbb{X}$ $H : [P, \vec{C}] : \mathbb{X}$

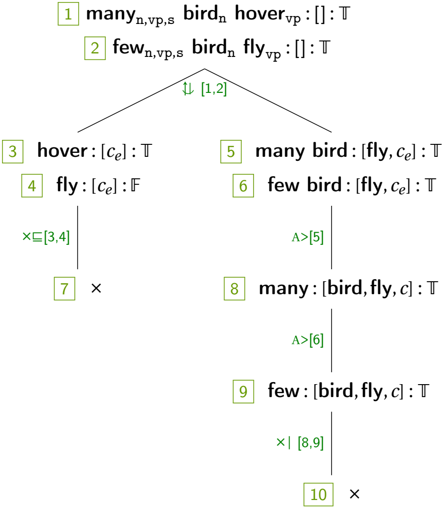
$G^{\uparrow}$ and $P = B$, or $H^{\uparrow}$ and $P = A$

⇵

$$GA : [\vec{C}] : \mathbb{X}$$
$$HB : [\vec{C}] : \mathbb{X}$$

$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$	$G : [P, \vec{C}] : \mathbb{X}$ $H : [P, \vec{C}] : \mathbb{X}$

$G^{\downarrow}$ and $P = B$, or $H^{\downarrow}$ and $P = A$



1 $\textbf{many}_{\mathrm{n,vp,s}}\ \textbf{bird}_{\mathrm{n}}\ \textbf{hover}_{\mathrm{vp}} : [\,] : \mathbb{T}$

2 $\textbf{few}_{\mathrm{n,vp,s}}\ \textbf{bird}_{\mathrm{n}}\ \textbf{fly}_{\mathrm{vp}} : [\,] : \mathbb{T}$

⇅ [1,2]

3 $\textbf{hover} : [c_e] : \mathbb{T}$

4 $\textbf{fly} : [c_e] : \mathbb{F}$

×⊑[3,4]

7 ×

5 $\textbf{many bird} : [\textbf{fly}, c_e] : \mathbb{T}$

6 $\textbf{few bird} : [\textbf{fly}, c_e] : \mathbb{T}$

A>[5]

8 $\textbf{many} : [\textbf{bird}, \textbf{fly}, c] : \mathbb{T}$

A>[6]

9 $\textbf{few} : [\textbf{bird}, \textbf{fly}, c] : \mathbb{T}$

×| [8,9]

10 ×

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*

Relation projection

Relation projection properties of F based on the relations |, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*
2. $(X \sqsubseteq Y) \rightarrow (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*
2. $(X \sqsubseteq Y) \rightarrow (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*
3. $(X \,|\, Y) \rightarrow (FX \,|\, FY)$ **skillful**, **most** N *multiplicative*

   skillful waiter | **skillful waitress**; **most** N **sleep** | **most** N **run**

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*
2. $(X \sqsubseteq Y) \rightarrow (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*
3. $(X \mid Y) \rightarrow (FX \mid FY)$ **skillful**, **most** N *multiplicative*
   skillful waiter | **skillful waitress**; **most** N **sleep** | **most** N **run**
4. $(X \mid Y) \rightarrow (FX \smile FY)$ **not**, **not every** N *anti-multiplicative*
   not run $\smile$ **not sleep**

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \to (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*
2. $(X \sqsubseteq Y) \to (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*
3. $(X \mid Y) \to (FX \mid FY)$ **skillful**, **most** N *multiplicative*
   skillful waiter | **skillful waitress**; **most** N **sleep** | **most** N **run**
4. $(X \mid Y) \to (FX \smile FY)$ **not**, **not every** N *anti-multiplicative*
   not run $\smile$ **not sleep**
5. $(X \smile Y) \to (FX \smile FY)$ **some**, **some** N *additive*
   some nonhuman $\smile$ **some animal**

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

1. $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*
2. $(X \sqsubseteq Y) \rightarrow (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*
3. $(X \mid Y) \rightarrow (FX \mid FY)$ **skillful**, **most** N *multiplicative*
   skillful waiter $|$ **skillful waitress**; **most** N **sleep** $|$ **most** N **run**
4. $(X \mid Y) \rightarrow (FX \smile FY)$ **not**, **not every** N *anti-multiplicative*
   not run $\smile$ **not sleep**
5. $(X \smile Y) \rightarrow (FX \smile FY)$ **some**, **some** N *additive*
   some nonhuman $\smile$ **some animal**
6. $(X \smile Y) \rightarrow (FX \mid FY)$ **not**, **no** N *anti-additive*
   no Ns **areAnimals** $|$ **no** Ns **areNonhumans**

Relation projection

Relation projection properties of F based on the relations $|$, $\smile$, and $\sqsubseteq$ ($\sqsupseteq$):

① $(X \sqsubseteq Y) \rightarrow (FX \sqsubseteq FY)$ **some**, **very** N *upward monotone*

② $(X \sqsubseteq Y) \rightarrow (FX \sqsupseteq FY)$ **every**, **no** N *downward monotone*

③ $(X \,|\, Y) \rightarrow (FX \,|\, FY)$ **skillful**, **most** N *multiplicative*

skillful waiter | **skillful waitress**; **most** N **sleep** | **most** N **run**

④ $(X \,|\, Y) \rightarrow (FX \smile FY)$ **not**, **not every** N *anti-multiplicative*

not run $\smile$ **not sleep**

⑤ $(X \smile Y) \rightarrow (FX \smile FY)$ **some**, **some** N *additive*

some nonhuman $\smile$ **some animal**

⑥ $(X \smile Y) \rightarrow (FX \,|\, FY)$ **not**, **no** N *anti-additive*

no Ns **areAnimals** | **no** Ns **areNonhumans**

7-10 $(X \,|\, Y) \rightarrow (FX \sqsubseteq FY)$; $(X \sqsubseteq Y) \rightarrow (FX \,|\, FY)$; $(X \smile Y) \rightarrow (FX \sqsubseteq FY)$; $(X \sqsubseteq Y) \rightarrow (FX \smile FY)$

Derivable monotone rules

$$\forall AB\big((A \sqsubseteq B) \to (FA \sqsubseteq FB)\big)$$

F↑⊑

$F^{\uparrow}A : [\vec{C}] : \mathbb{T}$
$F^{\uparrow}B : [\vec{C}] : \mathbb{F}$

$A : [\vec{D}] : \mathbb{T}$
$B : [\vec{D}] : \mathbb{F}$

$\vec{D}$ is fresh

Derivable monotone rules

$$\forall AB\big((A \sqsubseteq B) \rightarrow (FA \sqsubseteq FB)\big)$$

$$\begin{array}{c} \mathrm{F}{\uparrow}{\sqsubseteq} \\ \hline F^{\uparrow}A : [\vec{C}] : \mathbb{T} \\ F^{\uparrow}B : [\vec{C}] : \mathbb{F} \\ \hline A : [\vec{D}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} \\ \hline \vec{D} \text{ is fresh} \end{array}$$

is derived from

$$\begin{array}{c|c} \multicolumn{2}{l}{{\uparrow}{\sqsubseteq}} \\ \hline \multicolumn{2}{c}{G\ A : [\vec{C}] : \mathbb{T}} \\ \multicolumn{2}{c}{H\ B : [\vec{C}] : \mathbb{F}} \\ \hline A : [\vec{D}] : \mathbb{T} & G : [P, \vec{C}] : \mathbb{T} \\ B : [\vec{D}] : \mathbb{F} & H : [P, \vec{C}] : \mathbb{F} \\ \hline \multicolumn{2}{c}{G^{\uparrow} \text{ and } P = B, \text{ or } H^{\uparrow} \text{ and } P = A} \end{array}$$

and

$$\begin{array}{c} \times{\sqsubseteq} \\ \hline A : [\vec{C}] : \mathbb{T} \\ B : [\vec{C}] : \mathbb{F} \\ \hline \times \\ \hline A \sqsubseteq B \end{array}$$

Derivable monotone rules

$$\forall AB\big((A \sqsubseteq B) \rightarrow (FA \sqsubseteq FB)\big)$$

$\mathrm{F}{\uparrow}{\sqsubseteq}$
$F^{\uparrow}A : [\vec{C}] : \mathbb{T}$ $F^{\uparrow}B : [\vec{C}] : \mathbb{F}$
$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$
$\vec{D}$ is fresh

is derived from

${\uparrow}{\sqsubseteq}$	
$G\ A : [\vec{C}] : \mathbb{T}$ $H\ B : [\vec{C}] : \mathbb{F}$	
$A : [\vec{D}] : \mathbb{T}$ $B : [\vec{D}] : \mathbb{F}$	$G : [P, \vec{C}] : \mathbb{T}$ $H : [P, \vec{C}] : \mathbb{F}$
$G^{\uparrow}$ and $P = B$, or $H^{\uparrow}$ and $P = A$	

and

${\times}{\sqsubseteq}$
$A : [\vec{C}] : \mathbb{T}$ $B : [\vec{C}] : \mathbb{F}$
$\times$
$A \sqsubseteq B$

$$\forall AB\big((A \sqsubseteq B) \rightarrow (FB \sqsupseteq FA)\big)$$

$\mathrm{F}{\downarrow}{\sqsubseteq}$
$F^{\downarrow}A : [\vec{C}] : \mathbb{T}$ $F^{\downarrow}B : [\vec{C}] : \mathbb{F}$
$A : [\vec{D}] : \mathbb{F}$ $B : [\vec{D}] : \mathbb{T}$
$\vec{D}$ is fresh

is derived from

${\downarrow}{\sqsubseteq}$	
$G\ A : [\vec{C}] : \mathbb{T}$ $H\ B : [\vec{C}] : \mathbb{F}$	
$A : [\vec{D}] : \mathbb{F}$ $B : [\vec{D}] : \mathbb{T}$	$G : [P, \vec{C}] : \mathbb{T}$ $H : [P, \vec{C}] : \mathbb{F}$
$G^{\downarrow}$ and $P = B$, or $H^{\downarrow}$ and $P = A$	

and

${\times}{\sqsubseteq}$
$A : [\vec{C}] : \mathbb{T}$ $B : [\vec{C}] : \mathbb{F}$
$\times$
$A \sqsubseteq B$

Relation projection rules

1. $(X \mid Y) \to (FX \mid FY)$ *multiplicative*
2. $(X \mid Y) \to (FX \smile FY)$ *anti-multiplicative*
3. $(X \smile Y) \to (FX \smile FY)$ *additive*
4. $(X \smile Y) \to (FX \mid FY)$ *anti-additive*

F||

$FX : [\vec{C}] : \mathbb{T}$
$FY : [\vec{C}] : \mathbb{T}$

$X : [\vec{D}] : \mathbb{T}$
$Y : [\vec{D}] : \mathbb{T}$

F satisfies (1)

F|(

$FX : [\vec{C}] : \mathbb{F}$
$FY : [\vec{C}] : \mathbb{F}$

$X : [\vec{D}] : \mathbb{T}$
$Y : [\vec{D}] : \mathbb{T}$

F satisfies (2)

F((

$FX : [\vec{C}] : \mathbb{F}$
$FY : [\vec{C}] : \mathbb{F}$

$X : [\vec{D}] : \mathbb{F}$
$Y : [\vec{D}] : \mathbb{F}$

F satisfies (3)

F(|

$FX : [\vec{C}] : \mathbb{T}$
$FY : [\vec{C}] : \mathbb{T}$

$X : [\vec{D}] : \mathbb{F}$
$Y : [\vec{D}] : \mathbb{F}$

F satisfies (4)

Conclusion

- Simple type theory as a proxy to natural logic
- Tableau system for natural logic
- Monotonicity reasoning enabled by the monotone rules
- Making the tableau more Natural and Robust by:
    - Adding syntactic types
    - Introducing the function list:
      binary format → ternary format
    - Adding new (algebraic) rules

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