A Natural Proof System for Natural Language NPS4NL-1: Natural Language Inference

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ESSLLI 2019 in Rīga, Latvija



Introduction

About us

Natural language inference

Monotonicity & Natural logic

Conclusion



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Course is about

The following research questions are at the heart of the course:

- How are the meanings of natural language sentences related to each other?
- How to systematically reason with natural language sentences?
- How to get an explainable reasoning system?
- Can I use expressive but at the same time *friendly* meaning representations?

There will be many *natural* trees



Conclusion

Course is **NOT** about

- Machine leaning (and Artificial Neural Networks)
- Lexicalized formal compositional semantics: [[John]]@([[loves]]@[[Mary]]) = love(john,mary)
- Only toy examples
- Proving theorems about formal logics (but proving natural language theorems)

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A few prerequisites

- Syntax of first-order logic formulas
- Understanding semantics of first-order logic formulas: $\exists y \forall x. love(x, y) \rightarrow \forall x \exists y. love(x, y)$
- Some knowledge of λ -calculus
- Some knowledge of simply typed λ -calculus

Outcome of the course

In the end of the course you will know about:

- Challenges posed by Natural Language Inference
- Pros & Cons of logic-based methods wrt NLI
- Nitty-gritty details of theorem proving with a tableau system
- Doing semantics with higher-order logic
- How to account for a semantic phenomena in Natural Tableau
- How to use the Natural Tableau prover to solving inference problems

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Course in a nutshell



Topics per day

Mon Natural Language Inference

The task of NLI, monotonicity reasoning & natural logic (pros & cons)

Tue Semantic Tableau Method

Tableau systems for Propositional, First- and Higher-Order Logics

Wed Natural Tableau System

Lambda Logical Forms, Natural Tableau & tableau rules (part 1)

- Thu Wide-Coverage Theorem Prover for Natural Language Producing logical forms from syntactic trees, tableau rules (part 2)
 - Fri Natural Language Inference with Natural Theorem Prover Solving problems from NLI datasets, evaluation and analysis

Course web page: naturallogic.pro/Teaching/esslli19/

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Natural Language Understanding

Natural language understanding is one of the main problems of AI and NLP.

How to test whether a machine/program understands a natural language?

Use the Turing test. \blacktriangle Too expensive! $\heartsuit \heartsuit \heartsuit$



Juan Alberto Sánchez Margallo: https://commons.wikimedia.org/wiki/File:Test_de_Turing.jpg

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Natural Language Understanding

Natural language understanding is one of the main problems of AI and NLP.

How to test whether a machine/program understands a natural language?

Given a sentence, ask what does it mean/tell. A brown cat is lying on a mat

A1: A pet, which is brown, is lying

A2: A cat is on a mat

A3: There is an animal sleeping on a mat



 \blacksquare Evaluation of the answers requires a system that understands natural language. \heartsuit \heartsuit

Conclusion

Natural Language Understanding

Natural language understanding is one of the main problems of AI and NLP.

How to test whether a machine/program understands a natural language?

Given a sentence, ask a yes/no/dunno question about its meaning. A brown cat is lying on a yellow mat

Q1: Is the brown cat lying on the mat? [Yes]Q2: Is the mat soft? [Dunno]Q3: Is the cat jumping on the mat? [No]



▲This focuses on (long) questions and declarative sentences, and it is not straightforward to cover noun phrases and imperatives? ♥ A cat on the mat Feed the cat on the mat!

Conclusion

Natural Language Understanding

Natural language understanding is one of the main problems of AI and NLP.

How to test whether a machine/program understands a natural language?

Given two sentences S1 and S2, detect whether S1 entails S2.

S1: A brown cat is lying on a yellow mat

S2: There is an animal on a yellow-colored mat Answer: Yes

S1: The cat lying on a mat

S2: The cat rolling on a yellow mat

Answer: No



௴௴ Contrasting phrases of the same category, e.g., noun phrase, declarative sentences, questions, etc. ♥

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Recognizing Textual Entailment (2005-2013)

The task of Recognizing Textual Entailment (RTE) was introduced by [Dagan et al., 2005]:

- Textual entailment is defined as a *directional relationship* between pairs of text expressions, denoted by *T* (the entailing "Tex") and *H* (the entailed "Hypothesis"). We say that *T* entails *H* if humans reading *T* would typically infer that *H* is most likely true.
- An RTE task: given two texts, T (text) and H (hypothesis), detect textual entailment from T to H.
- The RTE1 to RTE3 challenges: binary classification
- The RTE4 to RTE8 challenges: 3-way classification with long texts

RTE problems

RTF2 GOLD: non-entailment

Drew Walker, NHS Tayside's public health director, said:

"It is important to stress that this is not a confirmed case of rabies."

A case of rabies was confirmed

RTE2

GOLD: entailment

About two weeks before the trial started, I was in Shapiro's office in Century City

Shapiro works in Century City

RTF2 GOLD: entailment

The drugs that slow down or halt Alzheimer's disease work best

the earlier you administer them

Alzheimer's disease is treated using drugs

Taken from [Dagan et al., 2013]

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Long before RTE: Aristotle's syllogisms

Aristotle's syllogisms (4th century BC):

- 256 RTE problems
- Text consists of two sentences
- 24(!) of the problems are entailment

OAO-3 GOLD: entailment Some cats have no tails All cats are mammals Some mammals have no tails

IAA-1GOLD: neutralSome vehicles are electricAll cars are vehiclesAll cars are electric

IAE-3GOLD: contradictionSome dogs have spotsAll dogs are mammalsNo mammals have spots

AAI-1GOLD: neutralAll canids are mammalsAll cupacabras are canidsSome cupacabras are mammals



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Just before RTE: FraCaS

The FraCaS test suite [Cooper et al., 1996]:

- 346 problems: a set of premises and a yes/no/dunno question
- Around half of the problems have multiple premises
- The problems are grouped based on the semantic phenomena: generalized quantifiers, plurals, ellipsis, adjectives, ...

FraCaS-26 GOLD: yes

Most Europeans are resident in Europe

All Europeans are people

All people who are resident in Europe can travel freely within Europe

Can most Europeans travel freely within Europe?

FraCaS-38 GOLD: unknown

No delegate finished the report.

Did any delegate finish the report on time?

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After RTE: Natural Language Inference

Natural Language Inference is a recent term for Recognizing Textual Entailment.



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Modern NLI

The Stanford NLI (SNLI) corpus [Bowman et al., 2015]:

- Large corpus: 570K premise-hypothesis pairs
- Tackling entity & event co-reference by grounding in images
- Premises are image captions and hypotheses are generated by crowd workers
- Premise-hypothesis pairs are annotated by 5 crowd workers with 3 labels

The Multi-Genre NLI (MultiNLI) corpus [Williams et al., 2018]:

- Large corpus: 433K premise-hypothesis pairs
- It is modeled on the SNLI corpus
- 10 genres: Fiction, Governmnet, Slate, Telephone, Travel, 9/11,...
- Used as a sentence encoder benchmark at RepEval 2019

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Examples from SNLI*

SNLI-3581451227.jpg#4r1c GOLD: contradiction^{1c} A little girl and boy after a wedding in a field the sail boat sank in the ocean

SNLI-475816542.jpg#2r1c GOLD: contradiction^{3c2e} A black and a brown dog are running toward the camera. A black and a brown cat are running toward the camera.

SNLI-4837051771.jpg#2r1n GOLD: neutral^{2e3n}

A small ice cream stand with two people standing near it.

Two people in line to buy icecream.

SNLI-2218907190.jpg#1r1e GOLD: entailment^{2c3e}

A dog begins to climb a brick staircase near plants.

A dog is going up the stairs.

*Arguable examples

Critical look at SNLI

In the test part of SNLI, 12.2% of problems get 2vs3 annotations.

Annotation artifacts inflate systems' performance [Poliak et al., 2018, Gururangan et al., 2018]:

- Hypothesis only baselines score strikingly high wrt the majority class baseline: 69.2% vs 33.8% accuracy
- animal, outdoors, and person often in entailment hypotheses
- tall, sad, and first often in neutral hypotheses
- cat, sleeping, and no often in contradiction hypotheses

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Smart black boxes

Author performance on test-SNLI (91.4%) is already suppressed by a deep neural network-based system (91.6%) [Liu et al., 2019]

Isaderboard Isaderboard

What does this mean?

Specially dedicated workshops: BlackboxNLP 2018 and 2019



Harder challenges aka Task-Independent Sentence Understanding:

Is GLUE leaderboard ■

■ SuperGLUE leaderboard

Explainable reasoning

Explainable reasoning is a feature associated with a white box systems:

- Explain entailment by providing some sort of proof or argument
- Explain contradiction by highlighting the incompatible cases
- Explain neutral relation by providing counterexamples for entailment and contradiction

e-SNLI – NLI with natural language explanations [Camburu et al., 2018]:

- For each labelled NLI problem, crowd workers gave explanations;
- Also the word relevant for explanations were highlighted;
- An NLI system needs to predict a label and an explanation;
- How to evaluate predicted explanation automatically? BLEU-score is a poor metric for this purpose.

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Examples from e-SNLI

SNLI-3581451227.jpg#4r1c GOLD: contradiction^{1c}

A little girl and boy after a wedding in a field

the sail boat sank in the ocean

A girl and boy are people, not a thing, as a sail boat is. You cannot be in a field and in the ocean at the same time

SNLI-475816542.jpg#2r1c GOLD: contradiction^{3c2e}

A black and a brown dog are running toward the camera.

A black and a brown cat are running toward the camera.

They refer to a dog, not a cat

The animal is either a cat or a dog

A dog cannot be a cat

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Examples from e-SNLI (II)

SNLI-4837051771.jpg#2r1n GOLD: neutral^{2e3n}

A small ice cream stand with two people standing near it.

Two people in line to buy icecream.

Being near a stand doesn't mean you have to buy anything Just because two people are standing near an ice cream stand, doesn't mean they are in line to buy ice cream

People who are standing near an ice cream stand are not always in line to buy ice cream

SNLI-2218907190.jpg#1r1e GOLD: entailment^{2c3e}

A dog begins to climb a brick staircase near plants.

A dog is going up the stairs.

Brick staircase is a paraphrase of stairs, and going up means to climb

Climbing implies going up

A DOG IS CLIMBING UP THE STAIRS

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Shortcomings of NLI systems

- Few NLI systems are able to reason over multiple premises.
- Most NLI systems do not use logic-based reasoning: poor at processing Booleans (e.g., *or*, *not*) and quantifiers (e.g., *every*, *no*).
 - P1: Most boxers have been knocked out
 - P2: All boxers are athletes

P3: All athletes who has been knocked out has a broken nose

C: Most boxers have a broken nose

- SOTA NLI systems are not explanatory (though pretty good).
- Most RTE systems can be fooled easily (i.e. not having high precision)

SICK-1745 GOLD:: neutral

- T: A man is pushing the buttons of a microwave
- H: A man is being pushed toward the buttons of a microwave

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Shortcomings of logic-based NLI systems

- Their logic is often not expressive enough to model some aspects of linguistics semantics: higher-order terms like generalized quantifiers (e.g., *few*, *most*) and subsective modifiers e.g. *competent*, *slowly*.
- Translation of linguistic semantics into formal logic is usually a complex and immense problem, e.g., NL text into first-order logic [Bos, 2008].
- After the translation, information about constituency and syntax is not available in a formal language while the information is often crucial for *shallow* reasoning, e.g., monotonicity reasoning.

SICK-8145 GOLD: entailment

- T: A woman in blue has a yellow ball in the mitt
- H: A woman in blue has a yellow ball in the hand

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How logic can be natural?

Natural logic is a hypothetical logic which is built in natural language and represents its integral part.

It is a theory about "the regularities governing the notion of a valid argument for reasoning in natural language" [Lakoff, 1970].

"Natural logic is a somewhat loose [...] term for [...] attempts [...] at describing basic patterns of human reasoning directly in natural language without the intermediate of some formal system" [van Benthem, 2008].

Natural logic is "the study of inference in natural language, done as close as possible to the surface forms" [Moss, 2010b].

Monotonicity reasoning

The most popular and success story of natural logic is monotonicity reasoning.

Monotonicitity reasoning is about replacing phrases in a premise in such a way that the obtained sentences are entailment of the premise.

GOLD: entailment

P: Every man who consumed alcohol devoured most snacks

H: Every young man who drank beer ate some snacks

GOLD: entailment $\begin{array}{l} \text{P: } 3 \times [s_3(x) = x + 3](2) \leq [p_3(x) = x^3] \big([m_4(x) = x \pmod{4}](7) \big) \\ \text{H: } 2 \times [s_1(x) = x + 1](1) \leq [p_4(x) = x^4] \big([m_8(x) = x \pmod{8}](7) \big) \end{array}$

Do you see similarity between these two entailment pairs? Now?

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Monotonicity reasoning in action

 $\begin{array}{l} \text{GOLD: entailment} \\ \hline \text{P: } 3 \times [s_3(x) = x + 3](2) &\leq [p_3(x) = x^3] \big([m_4(x) = x \pmod{4}](7) \big) \\ \hline \text{H: } 2 \times [s_1(x) = x + 1](1) &\leq [p_4(x) = x^4] \big([m_8(x) = x \pmod{8}](7) \big) \end{array}$



GOLD: entailment

P: Every man who consumed alcohol devoured most snacks

H: Every young man who drank beer ate some snacks

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NatLog [MacCartney, 2009]

First NLI system that introduced natural logic to the NLP community

- Semantic relation over lexical items
- Union of the semantic relations
- Projection of the relations

${ m symbol}^{10}$	name	example	set theoretic definition 11		
$x \equiv y$	equivalence	$couch \equiv sofa$	x = y		
$x \sqsubset y$	forward entailment	$crow \sqsubseteq bird$	$x \subset y$		
$x \sqsupset y$	reverse entailment	$Asian \ \Box \ Thai$	$x \supset y$		
$x \land y$	negation	$able \ \land \ unable$	$x\cap y=\emptyset\wedge x\cup y=U$		
$x \mid y$	alternation	$cat \mid dog$	$x\cap y=\emptyset\wedge x\cup y\neq U$		
$x \smile y$	cover	animal \sim non-ape	$x\cap y\neq \emptyset \wedge x\cup y=U$		
$x \ \# \ y$	independence	$hungry \ \# \ hippo$	(all other cases)		

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	\mathbf{pr}	oject	ivity	for 1	st ar	gume	ent	pro	ojecti	ivity	for 2	nd ar	gume	ent
quantifier	≡			^		\smile	#	≡			^		\smile	#
some	≡			\downarrow^{\dagger}	#	\downarrow^{\dagger}	#	≡			<u>_</u> †	#	_†	#
no	≡			†	#	†	#	≡			†	#	†	#
every	≡			‡	#	‡	#	≡			†	†	#	#
not every	≡		\Box	\smile^{\ddagger}	#	_‡	#	≡			<u>_</u> †	\downarrow^{\dagger}	#	#
at least two	≡			#	#	#	#	=			#	#	#	#
most	≡	#	#	#	#	#	#	=					#	#
exactly one	≡	#	#	#	#	#	#	=	#	#	#	#	#	#
$all \ but \ one$	≡	#	#	#	#	#	#	≡	#	#	#	#	#	#

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NatLog in action

P: John refused to move without blue jeans

H: John didn't dance without pants

Detect polarities of the words in the premise

	Sentence & Atomic edit	Lexical	Projected	Overall
(S0)	John refused to move without blue jeans			
	(E1) del(refused to)		1	
(S1)	John moved without blue jeans			
	(E2) ins(didn't)	^	^	
(S2)	John didn't moved without blue jeans			
	(E3) sub(move, dance)			
(S3)	John didn't dance without blue jeans			
	(E4) del(blue)			
(S4)	John didn't dance without jeans			
	(E5) sub(jeans, pants)			
(S5)	John didn't dance without pants			

$S0|S1 \bowtie S1^S2 \bowtie S2 S3 \bowtie S3 S4 \bowtie S4 S5 = S0 S5$

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Shortcomings of NatLog

• Cannot account for paraphrases:

John bought a car from Bill Bill sold a car to John

A student wrote an essay An essay was written by a student

• Weaker than first-order logic:

Not all bird fly Some birds does not fly

• The word-alignment and -substitution nature of reasoning falls short of processing multiple premises

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Related work

Other works on monotonicity reasoning and natural logic:

- First study of monotonicity reasoning as a formal calculus [Van Benthem, 1986, van Benthem, 1987, Sánchez-Valencia, 1991]
- Moving from syllogistic logics towards natural logic [Moss, 2010a]
- A tableau proof system for a fragment of natural logic [Muskens, 2010]
- Formal system for extended monotonicity reasoning [MacCartney and Manning, 2008, Icard, 2012, Icard and Moss, 2014]

Working systems:

- Monotonicity-based inference system for a fragment of English, operating on categorical grammar derivation trees [Fyodorov et al., 2003, Zamansky et al., 2006]
- Implementation of syllogistic logic with monotonicity [Eijck, 2005]
- Two implementations of extended syllogistic logics [Hemann et al., 2015]
- Natural language inference using polarity-marked parse trees [Hu et al., 2019]

Conclusion

The RTE/NLI task can be seen "as the best way of testing an NLP system's semantic capacity" [Cooper et al., 1996].

The NLI task is popular: many benchmarks and datasets

NLI systems comes with many flavours but we focus on logic-based ones

Monotonicity reasoning, the signature of natural logic

- Polarity marking
- String edit and word replacement reasoning

References I

Bos, J. (2008). Wide-coverage semantic analysis with boxer. In Bos, J. and Delmonte, R., editors, Semantics in Text Processing. STEP 2008 Conference Proceedings, Research in Computational Semantics, pages 277-286. College Publications. Bowman, S. R., Angeli, G., Potts, C., and Manning, C. D. (2015). A large annotated corpus for learning natural language inference. In Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing, pages 632-642, Lisbon, Portugal. Association for Computational Linguistics. Camburu, O.-M., Rocktäschel, T., Lukasiewicz, T., and Blunsom, P. (2018). e-snli: Natural language inference with natural language explanations. In Bengio, S., Wallach, H., Larochelle, H., Grauman, K., Cesa-Bianchi, N., and Garnett, R., editors, Advances in Neural Information Processing Systems 31, pages 9539-9549. Curran Associates. Inc. Cooper, R., Crouch, D., Eiick, J. V., Fox, C., Genabith, J. V., Jaspars, J., Kamp, H., Milward, D., Pinkal, M., Poesio, M., Pulman, S., Briscoe, T., Maier, H., and Konrad, K. (1996), FraCaS: A Framework for Computational Semantics, Deliverable D16, Dagan, I., Glickman, O., and Magnini, B. (2005). The pascal recognising textual entailment challenge. In Proceedings of the PASCAL Challenges Workshop on Recognising Textual Entailment. Dagan, I., Roth, D., Sammons, M., and Zanzotto, F. M. (2013). Recognizing Textual Entailment: Models and Applications. Synthesis Lectures on Human Language Technologies. Morgan & Claypool Publishers Eijck, J. V. (2005). Syllogistics = monotonicity + symmetry + existential import.

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References II



Fyodorov, Y., Winter, Y., and Francez, N. (2003). Order-based inference in natural logic. Logic Journal of the IGPL, 11(4):385–416.



Gururangan, S., Swayamdipta, S., Levy, O., Schwartz, R., Bowman, S., and Smith, N. A. (2018). Annotation artifacts in natural language inference data. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 2 (Short Papers)*, pages 107–112, New Orleans, Louisiana. Association for Computational Linguistics.



Hemann, J., Swords, C., and Moss, L. (2015). Two advances in the implementations of extended syllogistic logics. In Balduccini1, M., Mileo, A., Ovchinnikova, E., Russo, A., and Schüuller, P., editors, Joint Proceedings of the 2nd Workshop on Natural Language Processing and Automated Reasoning, and the 2nd International Workshop on Learning and Nonmonotonic Reasoning at LPNMR 2015, pages 1–14.



Hu, H., Chen, Q., and Moss, L. (2019). Natural language inference with monotonicity. In Proceedings of the 13th International Conference on Computational Semantics - Short Papers, pages 8–15, Gothenburg, Sweden. Association for Computational Linguistics.



Icard, T. F. (2012). Inclusion and exclusion in natural language. Studia Logica, 100(4):705-725.



Icard, T. F. and Moss, L. S. (2014). Recent progress on monotonicity. *Linguistic Issues in Language Technology*, 9.

Lakoff, G. (1970). Linguistics and natural logic. In Davidson, D. and Harman, G., editors, *Semantics of Natural Language*, volume 40 of *Synthese Library*, pages 545–665. Springer Netherlands.

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References III

Liu, X., He, P., Chen, W., and Gao, J. (2019). Multi-task deep neural networks for natural language
understanding. arXiv preprint arXiv:1901.11504.



MacCartney, B. (2009). Natural language inference. Phd thesis, Stanford University.



MacCartney, B. and Manning, C. D. (2008). Modeling semantic containment and exclusion in natural language inference. In Scott, D. and Uszkoreit, H., editors, *COLING*, pages 521–528.

Moss, L. S. (2010a). Logics for natural language inference. Expanded version of lecture notes from a course at ESSLLI 2010.

Moss, L. S. (2010b). Natural logic and semantics. In Aloni, M., Bastiaanse, H., de Jager, T., and

Schulz, K., editors, Logic, Language and Meaning: 17th Amsterdam Colloquium, Amsterdam, The Netherlands, December 16-18, 2009, Revised Selected Papers, pages 84–93. Springer Berlin Heidelberg, Berlin, Heidelberg.



Muskens, R. (2010). An analytic tableau system for natural logic. In Aloni, M., Bastiaanse, H., de Jager, T., and Schulz, K., editors, *Logic, Language and Meaning*, volume 6042 of *Lecture Notes in Computer Science*, pages 104–113. Springer Berlin Heidelberg.



Poliak, A., Naradowsky, J., Haldar, A., Rudinger, R., and Van Durme, B. (2018). Hypothesis only baselines in natural language inference. In *Proceedings of the Seventh Joint Conference on Lexical and Computational Semantics*, pages 180–191, New Orleans, Louisiana. Association for Computational Linguistics.



Sánchez-Valencia, V. (1991). Categorial grammar and natural reasoning. ILTI Publication Series for Logic, Semantics, and Philosophy of Language LP-91-08, University of Amsterdam.

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References IV



van Benthem, J. (1987). Meaning: Interpretation and inference. Synthese, 73(3):451-470.

van Benthem, J. (2008). A brief history of natural logic. In *Technical Report PP-2008-05*. Institute for Logic, Language & Computation.

Williams, A., Nangia, N., and Bowman, S. (2018). A broad-coverage challenge corpus for sentence understanding through inference. In Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long Papers), pages 1112–1122. Association for Computational Linguistics.

Zamansky, A., Francez, N., and Winter, Y. (2006). A 'natural logic' inference system using the lambek calculus. *Journal of Logic, Language and Information*, 15(3):273–295.