

# A Natural Proof System for Natural Language

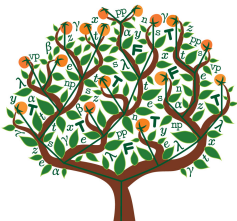
## Lecture 1: Natural Language Inference & Tableau Method

Lasha Abzianidze

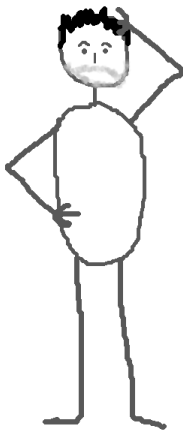


university of  
 groningen

TbiLLAI 2019 in Tbilisi, Georgia



# About me

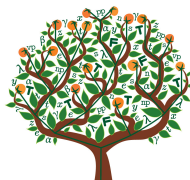


# About the course

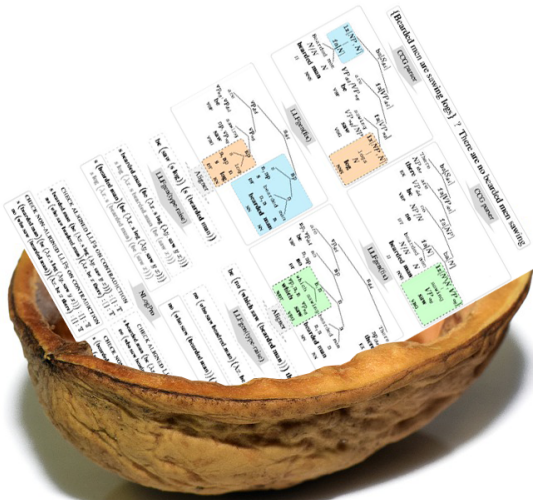
The following research questions are at the heart of the course:

- How are the meanings of natural language sentences related to each other?
- How to systematically reason with natural language sentences?
- How to get an explainable reasoning system?
- Can we use expressive but at the same time *friendly* meaning representations?

There will be a lot of *natural* trees



# Course in a nutshell



# Topics per day

**Mon** Natural Language Inference & Tableau Method

**Tue** 🛌

**Wed** 🛌

**Thu** 🚗

**Fri** Natural Tableau System

**Sat** Natural Language Inference with Natural Theorem Prover

Course web page: [naturallogic.pro/Teaching/tbillai19/](http://naturallogic.pro/Teaching/tbillai19/)

# Natural Language Understanding

Natural language understanding is one of the main problems of Artificial Intelligence and Natural Language Processing (NLP).

**How to test whether a machine/program understands a natural language?**

estimate semantic competence in a natural language

Given two sentences  $S1$  and  $S2$ , detect whether  $S1$  entails  $S2$ .

$S1$ : A brown cat is lying on a yellow mat

$S2$ : There is an animal on a yellow-colored mat

Answer: Yes

$S1$ : The cat is lying on a mat

$S2$ : The cat is rolling on a yellow mat

Answer: No

## Recognizing Textual Entailment (2005-2013)

The task of Recognizing Textual Entailment (RTE) was introduced by [Dagan et al., 2005]:

- **Textual entailment** is defined as a *directional relationship* between pairs of text expressions, denoted by  $T$  (the entailing “Text”) and  $H$  (the entailed “Hypothesis”). We say that  $T$  entails  $H$  if humans reading  $T$  would typically infer that  $H$  is most likely true.
- An RTE task: given two texts,  $T$  (text) and  $H$  (hypothesis), detect textual entailment from  $T$  to  $H$ .

# RTE problems

**RTE2** GOLD: non-entailment

Drew Walker, NHS Tayside's public health director, said:

“It is important to stress that this is not a confirmed case of rabies.”

---

A case of rabies was confirmed

**RTE2** GOLD: entailment

About two weeks before the trial started, I was in Shapiro's office  
in Century City

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Shapiro works in Century City

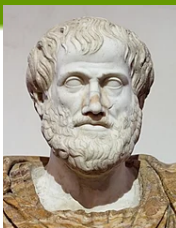
Taken from [Dagan et al., 2013]



# Long before RTE: Aristotle's syllogisms

Aristotle's syllogisms (4th century BC):

- 256 RTE problems
- Text consists of two sentences
- 24(!) of the problems are entailment



**OA0-3** GOLD: entailment

Some cats have no tails  
 All cats are mammals  
 -----  
 Some mammals have no tails

**IAE-3** GOLD: contradiction

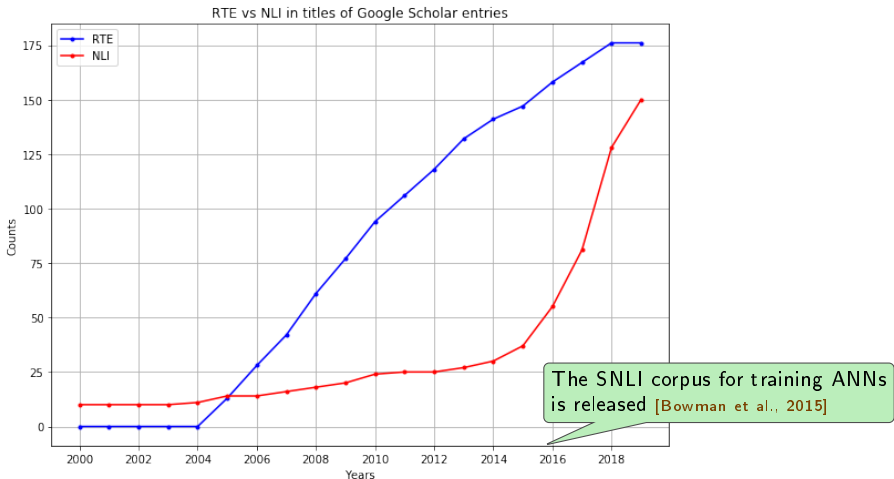
Some dogs have spots  
 All dogs are mammals  
 -----  
 No mammals have spots

**IAA-1** GOLD: neutral

Some vehicles are electric  
 All cars are vehicles  
 -----  
 All cars are electric

# After RTE: Natural Language Inference

Natural Language Inference is a recent term for Recognizing Textual Entailment.



# Modern NLI

The Stanford NLI (SNLI) corpus [Bowman et al., 2015]:

- Large corpus: 570K premise-hypothesis pairs
- Tackling entity & event co-reference by grounding in images
- Premises are image captions and hypotheses are generated by crowd workers
- Premise-hypothesis pairs are annotated by 5 crowd workers with 3 labels

# Examples from SNLI\*

SNLI-3581451227.jpg#4r1c

GOLD: contradiction<sup>1c</sup>

A little girl and boy after a wedding in a field  
 the sail boat sank in the ocean

SNLI-475816542.jpg#2r1c

GOLD: contradiction<sup>3c2n</sup>

A black and a brown dog are running toward the camera.  
 A black and a brown cat are running toward the camera.

SNLI-4837051771.jpg#2r1n

GOLD: neutral<sup>2e3n</sup>

A small ice cream stand with two people standing near it.  
 Two people in line to buy icecream.

\*Arguable examples

# Critical look at SNLI

In the test part of SNLI, 12.2% of problems get 2vs3 annotations.

Annotation artifacts inflate systems' performance

[Poliak et al., 2018, Gururangan et al., 2018]:

- Hypothesis only baselines score strikingly high wrt the majority class baseline: 69.2% vs 33.8% accuracy
- *animal*, *outdoors*, and *person* often in entailment hypotheses
- *tall*, *sad*, and *first* often in neutral hypotheses
- *cat*, *sleeping*, and *no* often in contradiction hypotheses

# Smart black boxes

Author performance on test-SNLI (91.4%) is already suppressed by a deep neural network-based system (91.6%) [Liu et al., 2019]

👉 [SNLI leaderboard](#)

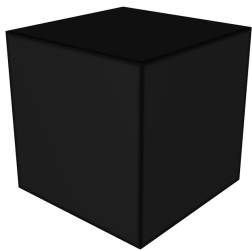
## What does this mean?

Specially dedicated workshops:  
[BlackboxNLP](#) 2018 and 2019

Harder challenges aka *Task-Independent Sentence Understanding*:

👉 [GLUE leaderboard](#)

👉 [SuperGLUE leaderboard](#)



# Explainable reasoning

**Explainable reasoning** is a feature associated with a white box systems:

- Explain entailment by providing some sort of proof or argument
- Explain contradiction by highlighting the incompatible cases
- Explain neutral relation by providing counterexamples for entailment and contradiction

**e-SNLI** – NLI with natural language explanations [Camburu et al., 2018]:

- Crowd workers gave explanations for labelled problems;
- Relevant words were highlighted;
- Systems need to predict a label and an explanation;

# Examples from e-SNLI

SNLI-3581451227.jpg#4r1c GOLD: contradiction<sup>1c</sup>

A little girl and boy after a wedding in a field

the sail boat sank in the ocean

A girl and boy are people, not a thing, as a sail boat is. You cannot be in a field and in the ocean at the same time

SNLI-475816542.jpg#2r1c GOLD: contradiction<sup>3c2e</sup>

A black and a brown dog are running toward the camera.

A black and a brown cat are running toward the camera.

They refer to a dog, not a cat

The animal is either a cat or a dog

A dog cannot be a cat



# Examples from e-SNLI (II)

SNLI-4837051771.jpg#2r1n

GOLD: neutral<sup>2e3n</sup>

A small ice cream stand with two people standing near it.

Two people **in line** to **buy** icecream.

Being near a stand doesn't mean you have to buy anything

Just because two people are standing near an ice cream stand, doesn't mean they are in line to buy ice cream

People who are standing near an ice cream stand are not always in line to buy ice cream

# Shortcomings of NLI systems

- Few NLI systems are able to reason over **multiple premises**.
- Most NLI systems do not use **logic-based reasoning**: poor at processing Booleans (e.g., *or*, *not*) and quantifiers (e.g., *every*, *no*).  
 P1: Most boxers have been knocked out  
 P2: All boxers are athletes  
 P3: All athletes who has been knocked out has a broken nose  


---

 C: Most boxers have a broken nose
- SOTA NLI systems are not explanatory (though pretty good).
- Most RTE systems can be **fooled easily** (i.e. not having high precision)

SICK-1745 GOLD:: neutral

T: A man is pushing the buttons of a microwave

H: A man is being pushed toward the buttons of a microwave

## Shortcomings of logic-based NLI systems

- Their logic is often **not expressive enough** to model some aspects of linguistics semantics: higher-order terms like generalized quantifiers (e.g., *few*, *most*) and subsecutive modifiers e.g. *competent*, *slowly*.
- **Translation** of linguistic semantics into formal logic is usually a **complex and immense problem**, e.g., NL text into first-order logic [Bos, 2008].
- After the translation, information about **constituency and syntax is not available** in a formal language while the information is often crucial for *shallow* reasoning, e.g., monotonicity reasoning.

SICK-8145 GOLD: entailment

T: A woman in blue has a yellow ball in the mitt

H: A woman in blue has a yellow ball in the hand

# How logic can be natural?

**Natural logic** is a hypothetical logic which is built in natural language and represents its integral part.

It is a theory about “the regularities governing the notion of a valid argument for reasoning in natural language” [Lakoff, 1970].

“Natural logic is a somewhat loose [...] term for [...] attempts [...] at describing basic patterns of human reasoning directly in natural language without the intermediate of some formal system”

[van Benthem, 2008].

Natural logic is “the study of inference in natural language, done as close as possible to the surface forms” [Moss, 2010b].

# Monotonicity reasoning

The most popular and success story of natural logic is **monotonicity reasoning**.

Monotonicity reasoning is about replacing phrases in a premise in such a way that the obtained sentences are entailment of the premise.

GOLD: entailment

P: Every man who consumed alcohol devoured most snacks

H: Every young man who drank beer ate some snacks

GOLD: entailment

P:  $3 \times [s_3(x) = x + 3](2) \leq [p_3(x) = x^3]([m_4(x) = x \pmod{4}](7))$

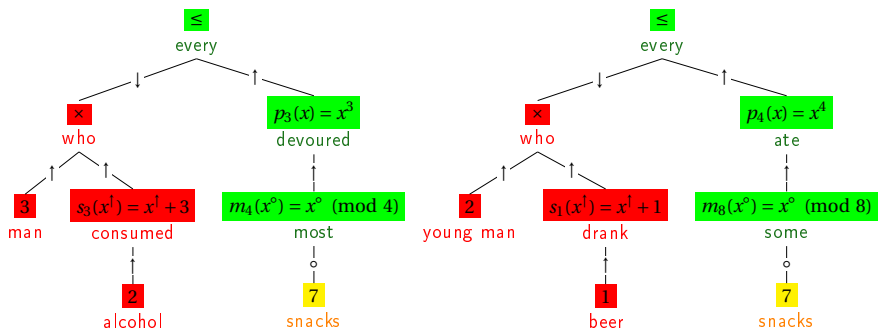
H:  $2 \times [s_1(x) = x + 1](1) \leq [p_4(x) = x^4]([m_8(x) = x \pmod{8}](7))$

# Monotonicity reasoning in action

GOLD: entailment

P:  $3 \times [s_3(x) = x + 3](2) \leq [p_3(x) = x^3]([m_4(x) = x \pmod{4}](7))$

H:  $2 \times [s_1(x) = x + 1](1) \leq [p_4(x) = x^4]([m_8(x) = x \pmod{8}](7))$



GOLD: entailment

P: Every **man** who **consumed alcohol** **devoured most** snacks

H: Every **young man** who **drank beer** **ate some** snacks

## Related work

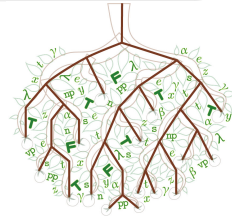
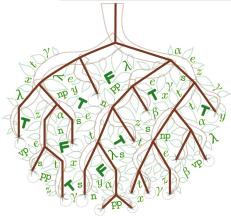
Other works on monotonicity reasoning and natural logic:

- First study of monotonicity reasoning as a formal calculus [Van Benthem, 1986, van Benthem, 1987, Sánchez-Valencia, 1991]
- Moving from syllogistic logics towards natural logic [Moss, 2010a]
- A tableau proof system for a fragment of natural logic [Muskens, 2010]
- Formal system for extended monotonicity reasoning [MacCartney and Manning, 2008, Icard, 2012, Icard and Moss, 2014]

Working systems:

- Monotonicity-based inference system for a fragment of English, operating on categorical grammar derivation trees [Fyodorov et al., 2003, Zamansky et al., 2006]
- Implementation of syllogistic logic with monotonicity [Eijck, 2005]
- Two implementations of extended syllogistic logics [Hemann et al., 2015]
- Natural language inference using polarity-marked parse trees [Hu et al., 2019]

# Semantic Tableau Method





# Logic & proof systems

Logic consists of four components:

- Intuitive non-formal motivation
- Syntax of formulas: well-formed formulas vs ill-formed ones
- Semantics associated with the formulas
- Some type of proof calculus

A proof calculus/system:

- employed to systematically capture valid formulas and arguments
- is a syntactic game: there are legal and illegal moves
- comes in several flavours
- is usually a sound and complete

# Semantic tableau method

A **semantic tableau method** [Beth, 1955, Hintikka, 1955] is a proof procedure for formal logics that checks formulas with truth constraints:

**Input:** A set of signed formulas

$$P_1 : \mathbb{T}, \dots, P_m : \mathbb{T}, Q_1 : \mathbb{F}, \dots, Q_n : \mathbb{F}$$

**Output:** some or no model satisfying the truth constraints on the formulas  
 ☞ A model search problem

# Prove or refute

Whenever it rains, the roof leaks

How to verify truth of this statement?

Show that:

- In **every** situation it is true

Approval route

Check **every** situation when it rains and show the roof leaking

- In **some** situation it is **not** true

Refutation route

Find **some** situation when it rains and the roof isn't leaking

# Proving by failing to refute

A tableau method tries to refute statement in order to prove it:

- 1 Given  $P_1, \dots, P_m \models Q$  to prove
- 2 Try to refute  $P_1, \dots, P_m \models Q$ 
  - 1 Build the counterexample:  $P_1 : \mathbb{T}, \dots, P_m : \mathbb{T}, Q : \mathbb{F}$
  - 2 Try to satisfy the counterexample
- 3 If refutation succeeded,  $P_1, \dots, P_m \models Q$  is disproved
- 4 Otherwise  $P_1, \dots, P_m \models Q$  is proved

# Propositional tableau method (signed version)

Prove:  $P \wedge Q \models Q \wedge \neg P$

Counterexample:  $P \wedge Q : \text{T}, Q \wedge \neg P : \text{F}$

Propositional tableau rules:

|                         |
|-------------------------|
| $\wedge_{\text{T}}$     |
| $X \wedge Y : \text{T}$ |
| $X : \text{T}$          |
| $Y : \text{T}$          |

|                         |
|-------------------------|
| $\wedge_{\text{F}}$     |
| $X \wedge Y : \text{F}$ |
| $X : \text{F}$          |
| $Y : \text{F}$          |

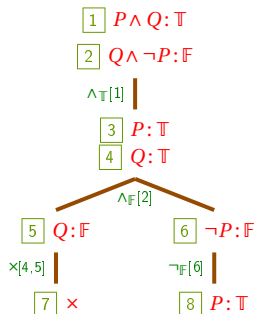
|                       |
|-----------------------|
| $\vee_{\text{T}}$     |
| $X \vee Y : \text{T}$ |
| $X : \text{T}$        |
| $Y : \text{T}$        |

|                       |
|-----------------------|
| $\vee_{\text{F}}$     |
| $X \vee Y : \text{F}$ |
| $X : \text{F}$        |
| $Y : \text{F}$        |

|                     |
|---------------------|
| $\neg_{\text{F}}$   |
| $\neg X : \text{F}$ |
| $X : \text{T}$      |

|                     |
|---------------------|
| $\neg_{\text{T}}$   |
| $\neg X : \text{T}$ |
| $X : \text{F}$      |

|                |
|----------------|
| $\times$       |
| $X : \text{T}$ |
| $X : \text{F}$ |
| $\times$       |



A situation supporting  
a counterexample:  $P : \text{T}, Q : \text{T}$

# Closed tableau

Prove:  $\neg(P \wedge Q) \models \neg P \vee \neg Q$  **Proved!**

Counterexample:  $\neg(P \wedge Q) : \top, \neg P \vee \neg Q : \text{F}$

Propositional tableau rules:

|                     |
|---------------------|
| $\wedge_T$          |
| $X \wedge Y : \top$ |
| $X : \top$          |
| $Y : \top$          |

|                         |
|-------------------------|
| $\wedge_F$              |
| $X \wedge Y : \text{F}$ |
| $X : \text{F}$          |
| $Y : \text{F}$          |

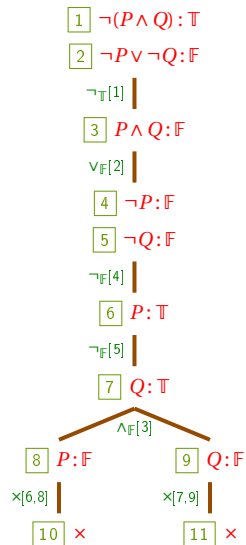
|                   |
|-------------------|
| $\vee_T$          |
| $X \vee Y : \top$ |
| $X : \top$        |
| $Y : \top$        |

|                       |
|-----------------------|
| $\vee_F$              |
| $X \vee Y : \text{F}$ |
| $X : \text{F}$        |
| $Y : \text{F}$        |

|                     |
|---------------------|
| $\neg_F$            |
| $\neg X : \text{F}$ |
| $X : \top$          |

|                 |
|-----------------|
| $\neg_T$        |
| $\neg X : \top$ |
| $X : \text{F}$  |

|                |
|----------------|
| $\times$       |
| $X : \top$     |
| $X : \text{F}$ |
| $\times$       |



# Different proof strategy

Prove:  $\neg(P \wedge Q) \models \neg P \vee \neg Q$  **Prover!**

Counterexample:  $\neg(P \wedge Q) : \text{T}, \neg P \vee \neg Q : \text{F}$

Propositional tableau rules:

|                         |
|-------------------------|
| $\wedge_{\text{T}}$     |
| $X \wedge Y : \text{T}$ |
| $X : \text{T}$          |
| $Y : \text{T}$          |

|                         |
|-------------------------|
| $\wedge_{\text{F}}$     |
| $X \wedge Y : \text{F}$ |
| $X : \text{F}$          |
| $Y : \text{F}$          |

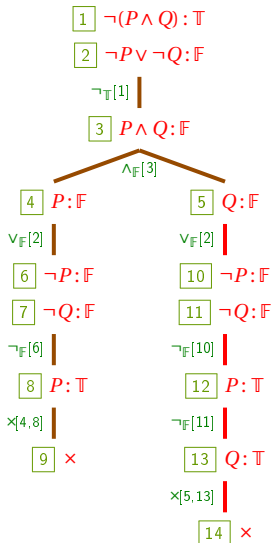
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| $\vee_{\text{T}}$     |
| $X \vee Y : \text{T}$ |
| $X : \text{T}$        |
| $Y : \text{T}$        |

|                       |
|-----------------------|
| $\vee_{\text{F}}$     |
| $X \vee Y : \text{F}$ |
| $X : \text{F}$        |
| $Y : \text{F}$        |

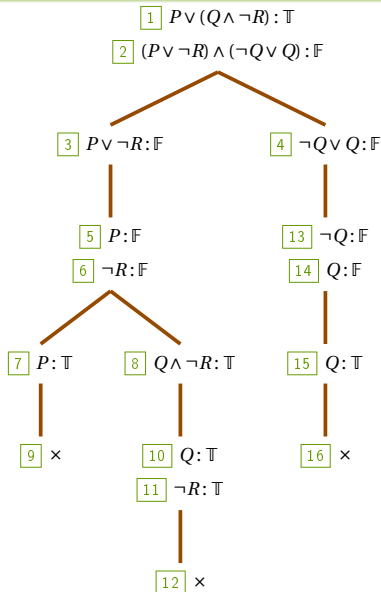
|                     |
|---------------------|
| $\neg_{\text{F}}$   |
| $\neg X : \text{F}$ |
| $X : \text{T}$      |

|                     |
|---------------------|
| $\neg_{\text{T}}$   |
| $\neg X : \text{T}$ |
| $X : \text{F}$      |

|                |
|----------------|
| $\times$       |
| $X : \text{T}$ |
| $X : \text{F}$ |
| $\times$       |



# Tableau exercise



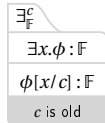
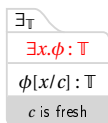


# Quiz

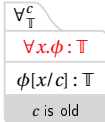
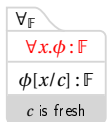
- 1 If propositional formula  $\phi$  is built up from  $n$  Boolean connectives, at most how many rule applications will be applicable to the tableau started with  $\phi:\top$ ?
- 2 ... started with  $\phi:\mathbb{F}$ ?
- 3 Can you think of tableau rules for  $\rightarrow_{\top}$  and  $\rightarrow_{\mathbb{F}}$ ?

# Rules for quantifiers

## Rules for $\exists$ :



## Rules for $\forall$ :



**!** Dangerous zone!

1  $\forall x.\exists y.\text{love}(x,y) : T$

2  $\forall z.\text{love}(z,z) : F$

$\forall_F[2]$

3  $\text{love}(c,c) : F$

$\forall_T^c[1]$

4  $\exists y.\text{love}(c,y) : T$

$\exists_T^c[4]$

5  $\text{love}(c,d) : T$

$\forall_T^d[1]$

6  $\exists y.\text{love}(d,y) : T$

⋮

# Non-empty domain

Rules for  $\exists$ :

|                         |
|-------------------------|
| $\exists_T$             |
| $\exists x.\phi : \top$ |
| $\phi[x/c] : \top$      |
| <i>c</i> is fresh       |

|                         |
|-------------------------|
| $\exists_F^c$           |
| $\exists x.\phi : \top$ |
| $\phi[x/c] : \top$      |
| <i>c</i> is old         |

1  $\forall x.(\text{run}(x) \wedge \neg\text{run}(x)) : \top$

Non-empty domain constraint:  
you can always have an entity

Rules for  $\forall$ :

|                         |
|-------------------------|
| $\forall_F$             |
| $\forall x.\phi : \top$ |
| $\phi[x/c] : \top$      |
| <i>c</i> is fresh       |

|                         |
|-------------------------|
| $\forall_T^c$           |
| $\forall x.\phi : \top$ |
| $\phi[x/c] : \top$      |
| <i>c</i> is old         |

# Conclusion: Natural Language Inference

The RTE/NLI task can be seen “as the best way of testing an NLP system’s semantic capacity” [Cooper et al., 1996].

The NLI task is popular: many benchmarks and datasets

NLI systems comes with many flavours but we focus on logic-based ones

Monotonicity reasoning, the signature of natural logic

- Polarity marking
- String edit and word replacement reasoning

# Conclusion: Tableau Method

- A semantic tableau method  
“today [it is] one of the most popular, since it appears to bring together the proof-theoretical and the semantical approaches to the presentation of a logical system and is also very intuitive. In many universities it is the style first taught to students.”  
[D’Agostino et al., 1999].
- Propositional tableau system: when applying a rule to a tableau entry, remember to do so for each branch it sits on.
- Dangerous zone: First-order logic tableau might not terminate

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